Solution to Problem 18

Problem. To solve an equation $a \cdot y^6 + b \cdot y^3 + c = 0$, a natural idea is to introduce a new variable $z = y^3$ for which we will have a quadratic equation – an equation that we know how to solve. Describe the resulting reductions in general terms: what is $C(x, y)$, what is $C'(x', y')$, what is U_1, U_2, and U_3.

Solution. Here $x = (a, b, c)$, $C(x, y)$ is the desired property $a \cdot y^6 + b \cdot y^3 + c = 0$. For the problem to which we reduce, we have $x' = (a, b, c)$, $y' = z$, and the property $C'(x', y')$ is $a \cdot z^2 + b \cdot z + c = 0$.

Here, $U_1(a, b, c) = (a, b, c)$, $U_2(y') = \sqrt[3]{y'}$, and $U_3(y) = y^3$.