Solution to Problem 31

Problem. Show how to compute the "or" of 14 boolean values in parallel if we have unlimited number of processors. How many processors do we need and how much time will the computation take? Why do we need parallel processing in the first place?

Solution. Suppose that we are given 14 boolean values $x_1, \ldots, x_{14}$, and we want to compute their disjunction ("or"). Then:

- at the first moment of time:
  - the first computer computes $x_1 \vee x_2$,
  - the second computer computes $x_3 \vee x_4$,
  - the third computer computes $x_5 \vee x_6$,
  - the forth computer computes $x_7 \vee x_8$,
  - the fifth computer computes $x_9 \vee x_{10}$;
  - the sixth computer computes $x_{11} \vee x_{12}$;
  - the seventh computer computes $x_{13} \vee x_{14}$.
- at the second moment of time:
  - the first computer computes $x_1 \vee x_2 \vee x_3 \vee x_4$ as
    $$(x_1 \vee x_2) \vee (x_3 \vee x_4);$$
  - the second computer computes $x_5 \vee x_6 \vee x_7 \vee x_8$ as
    $$(x_5 \vee x_6) \vee (x_7 \vee x_8);$$
  - the third computer computes $x_9 \vee x_{10} \vee x_{11} \vee x_{12}$ as
    $$(x_9 \vee x_{10}) \vee (x_{11} \vee x_{12}).$$
- at the third moment of time:
  - the first computer computes
    $$x_1 \vee x_2 \vee x_3 \vee x_4 \vee x_5 \vee x_6 \vee x_7 \vee x_8$$
    as
    $$(x_1 \vee x_2 \vee x_3 \vee x_4) \vee (x_5 \vee x_6 \vee x_7 \vee x_8);$$

– the second computer computes

$$x_9 \vee x_{10} \vee x_{11} \vee x_{12} \vee x_{13} \vee x_{14}$$

as

$$(x_9 \vee x_{10} \vee x_{11} \vee x_{12}) \vee (x_{13} \vee x_{14}).$$

- at the fourth moment of time, the first computer computes the desired value

$$x_1 \vee x_2 \vee x_3 \vee x_4 \vee x_5 \vee x_6 \vee x_7 \vee x_8 \vee x_9 \vee x_{10} \vee x_{11} \vee x_{12} \vee x_{13} \vee x_{14}$$

as

$$(x_1 \vee x_2 \vee x_3 \vee x_4 \vee x_5 \vee x_6 \vee x_7 \vee x_8) \vee (x_9 \vee x_{10} \vee x_{11} \vee x_{12} \vee x_{13} \vee x_{14}).$$

These computations require 7 computers and 4 moments of time.

In general, parallel computations are needed to speed up computations. Without parallelism, we would need 14 moments of time to compute the desired value.