

Solution to Homework 32

Problem. If we take into account communication time, how fast can you compute

$$1 \cdot \frac{1}{2} \cdot \dots \cdot \frac{1}{n}$$

in parallel?

Solution. According to Section 2 of the corresponding lecture, if we can solve the problem in parallel in time T_{parallel} , then we can also solve it sequentially in time $T_{\text{sequential}} \leq c \cdot T_{\text{parallel}}^4$. For computing the above product, the smallest possible time is n : we need $n - 1$ multiplications and 1 division: $1/(1 \cdot \dots \cdot n)$. Thus, $n \leq c \cdot T_{\text{parallel}}^4$. Dividing both sides by c , we get $c^{-1} \cdot n \leq T_{\text{parallel}}^4$, hence

$$T_{\text{parallel}} \geq C \cdot n^{1/4} = C \cdot \sqrt[4]{n}.$$

This is faster than the sequential time n , but much slower than the time $\text{const} \cdot \log(n)$ that we would have if we ignored communication time.