Theory of Computation, Spring 2026, Test 3

Problem 1. Explain where and how, in the analysis of how much parallelization can speed up computations, we use the two physical assumptions: that all speeds are bounded by the speed of light c, and that the volume of a sphere is proportional to the cube of its radius.

Problem 2. Use the general algorithm to translate the formula

$$(p \vee q \vee r \vee \neg s) \,\&\, (s \vee t)$$

into 3-CNF.

Problem 3–4. Reduce the satisfiability problem for the formula

$$(\neg a \vee \neg b \vee c) \,\&\, (a \vee \neg b)$$

to:

- 3-coloring,
- clique,
- subset sum problem, and
- interval computations.

In all these reductions, explain what will correspond to $a = T$, $b = F$, and $c = T$.

Problem 5. Show how to compute the minimum of 9 integer values in parallel if we have an unlimited number of processors and we can ignore communication time. Why do we need parallel processing in the first place? If we take communication time into account, how much time do we need to compute the minimum n values? What is NC? Give an example of a P-complete problem.

Problem 6. What can you say about the Kolmogorov complexity of the following string: 01100110... in which 0110 is repeated 1,000 times.

Turn over, please

Problem 7. Suppose that we have a probabilistic algorithm that gives a correct answer 3/4 of the time. How many times do we need to repeat this algorithm to reduce probability of error to at most 5%? Give an example of a probabilistic algorithm. Explain why we need probabilistic algorithms in the first place.

Problem 8. Use the variable-elimination algorithm for checking satisfiability of the following 2-SAT formula:

$$(\neg a \vee \neg c) \,\&\, (\neg b \vee c) \,\&\, (a \vee \neg b) \,\&\, (\neg b \vee \neg c).$$

Find all solutions. Start by eliminating a, then, if needed, eliminate b.

Problem 9. How is the NOR operation $f(x_1, x_2) = \neg(x_1 \vee x_2)$ represented in quantum computing? Provide a general formula and explain it on the example when x_1 is true and both x_2 and the auxiliary variable are false.

Problem 10. Why do we need to study recursively enumerable (r.e.) sets? Is the union of three r.e. sets still r.e.? If yes, prove it, if no, provide a counterexample.