Solution to Problem 13

**Task.** Use the interval-based optimization algorithm to locate the maximum of the function $f(x) = -2x + 4x^2$ on the interval $[0, 0.4]$. Divide this interval into two, then divide the remaining intervals into two again, etc. At each iteration, dismiss subintervals where maximum cannot be attained. Stop when you get intervals of width 0.1.

**Solution.** Let us first divide this interval into two subintervals: $[0, 0.2]$ and $[0.2, 0.4]$. On the first interval, the range of the derivative $-2 + 8x$ is equal to $-2 + 8 \cdot [0, 0.2] = [-2, 0.16]$. All these values are negative, so the function is decreasing on this subinterval.

- Its smallest value is attained when $x_1$ is the largest $x_1 = 0.2$ and is equal to $f(0.2) = -2 \cdot 0.2 + 4 \cdot 0.2^2 = -0.4 + 0.16 = -0.24$.

- Its largest value is attained when $x_1$ is the smallest $x_1 = 0$ and is equal to $f(0) = 0$.

Thus, the range of the function on the first subinterval is $[-0.24, 0]$.

For the second subinterval, the range of the derivative is equal to $-2 + 8 \cdot [0.2, 0.4] = -2 + [1.6, 3.2] = [-0.4, 1.2]$. This range contains both positive and negative values, so we have to use the centered form. For this subinterval, the midpoint is $\bar{x} = 0.3$. At this point, the value of the function is $f(0.3) = -2 \cdot 0.3 + 4 \cdot 0.3^2 = -0.6 + 0.36 = -0.24$. The half-widths of this subinterval is $\Delta = 0.1$. Thus, the centered form leads to the following enclosure for the range:

$$f(\bar{x}) + f'(\bar{x}) \cdot [-\Delta, \Delta] = -0.24 + [-0.4, 1.2] \cdot [-0.1, 0.1] = -0.24 + [-0.12, 0.12] = [-0.36, -0.12].$$
The largest value from this range is smaller than one of the values $f(0) = 0$, which means that the maximum cannot be attained on the second subinterval.

Thus, the maximum is attained on the first subinterval. Since on the first subinterval, the function is decreasing, its maximum there is attained at the smallest value $x = 0$. Thus, the maximum of the function $f(x)$ on the whole interval $[0, 0.4]$ is attained when $x = 0$. 
