Final Exam for Interval Class, Fall 2023

**Problem 1.** Why do we need data processing? Why do we need interval computations? What is the main problem of interval computations?

**Problem 2–4.** Use several methods to estimate the range of the function \( x^2 + 2x + 1 \) on the interval \([-2, 0]\).

- Use calculus.
- Why cannot we use calculus to find the range for any number of inputs?
- Use linearization method based on the actual derivative.
- Use linearization method based on numerical differentiation.
- Use straightforward interval computations.
- Use full interval computations: monotonicity checking, centered form, and bisection (one bisection step is sufficient).

**Problem 5.** Use the interval-based optimization algorithm to locate the maximum of the function \( x^2 + 2x + 1 \) on the interval \([-2, 0]\). Divide this interval into two, then divide the remaining intervals into two again, etc. At each iteration, dismiss subintervals where maximum cannot be attained. Stop when you get intervals of width 0.5.

**Problem 6.** Use constraints methods to solve the systems \( x_1 + 2x_2 = 1 \) and \( x_1 - 2x_2 = 0 \) for \( x_1, x_2 \in [0, 1] \). Use one bisection step.

**Problem 7.** If we have \( \mu(1) = 0.6 \) and \( \mu(3) = 0 \), what value shall we assign to \( \mu(2) \)? Use linear interpolation.

**Problem 8.** If for \( \alpha = 0.7 \), the \( \alpha \)-cut of \( x \) is \([-2, 0]\), and \( y = x^2 + 2x + 1 \), what is the corresponding \( \alpha \)-cut of \( y \)? Explain your answer.