Solution to Problem 11

Task. Use the full interval algorithm – monotonicity, centered form, and bisection (one iteration) – to estimate the range of the function from Problem 1 when x is in the interval $[-4, 0]$.

Solution. The only difference from Problem 9 is that on the subinterval $[-2, 0]$ on which the function is not monotonic, we should use centered form instead of straightforward interval computations.

The midpoint of this interval is $\widetilde{x} = \dfrac{-2+0}{2} = -1$, and its half-width is $\Delta = \dfrac{0-(-2)}{2} = 1$. Here,

$$f(\widetilde{x}) = f(-1) = (4 + (-1)) \cdot (2 - (-1)) = 3 \cdot 4 = 9.$$

We know, from the solution to Problem 9, that the range of the derivative on this subinterval is $[-2, 2]$. Thus, the centered form leads to the following enclosure for the range:

$$9 + [-2, 2] \cdot [-1, 1] = 9 + [-2, 2] = [7, 11].$$

The resulting range is then equal to the union of the ranges corresponding to both subintervals:

$$[0, 8] \cup [7, 11] = [0, 11].$$