Solution to Problem 12

Task. Use the full interval algorithm – monotonicity, centered form, and bisection (one iteration) – to estimate the range of the function from Problem 2 when x_1 is in the interval $[0, 6]$ and x_2 is in the interval $[-6, 6]$.

Solution. The main difference from Problem 10 is that on the sub-box where $x_1 \in [0, 3]$ and $x_2 \in [-6, 6]$ (and where there is no monotonicity), we need to use centered form instead of the straightforward interval computations.

We already know, from the solution to Problem 10, that for the x_2-interval, we gave $\widetilde{x}_2 = 0$ and $\Delta_2 = 6$. Here:

$$\widetilde{x}_1 = \frac{0+3}{2} = 1.5 \text{ and } \Delta_1 = \frac{3-0}{2} = 1.5.$$

Here,

$$f(\widetilde{x}_1, \widetilde{x}_2) = f(1.5, 0) = (2 - 1.5) \cdot (4 + 0) = 0.5 \cdot 4 = 2.$$

The ranges of the derivatives have been computed in the solution to Problem 10: it is $[-10, 2]$ for derivative with respect to x_1 and $[-1, 2]$ for the derivative with respect to x_2. Thus, the centered form leads to the following enclosure for the range:

$$2 + [-10, 2] \cdot [-1.5, 1.5] + [-1, 2] \cdot [-6, 6] = [2, 2] + [-15, 15] + [-12, 12] =$$

$$[-13, 17] + [-12, 12] = [-25, 29].$$

The find the range over the whole box, we take the union of the ranges over two sub-boxes:

$$[-25, 29] \cup [-40, 8] = [-40, 29].$$