Solution to Problem 15

Task. Use the constraints method to solve the following two problems:

- find x_1 from the interval $[0, 4]$ and x_2 from the interval $[-2, 0]$, for which

$$x_1 + x_2 = 0 \text{ and } x_1 \cdot x_2 = -1.0;$$

- find x_1 and x_2 from the same intervals for which

$$x_1 + x_2 = 0 \text{ and } x_1 \cdot x_2 = -20.0.$$

Comment: once we conclude that the variables belong to narrower intervals, we bisect these narrower intervals, and not the original wider ones.

First system of equations. Based on the equations, we form the following four rules:

1. $x_1 \mapsto x_2 = -x_1$;
2. $x_2 \mapsto x_1 = -x_2$;
3. $x_1 \mapsto x_2 = -4/x_1$;
4. $x_2 \mapsto x_1 = -4/x_2$.

We start with $x_1 \in \mathbf{x}_1 = [0, 4]$ and $x_2 \in \mathbf{x}_2 = [-2, 0]$. We apply Rules 1, 2, 3, 4, then again 1, 2, 3, 4, etc. If we get stuck, we bisect one of the intervals.

- Rule 1 leads to $\mathbf{x}_2 = [-2, 0] \cap ((-1) \cdot [1, 4]) = [-2, 0] \cap [-4, 0] = [-2, -1]$.
- Rule 2 leads to $\mathbf{x}_1 = [0, 4] \cap ((-1) \cdot [-2, 0]) = [0, 4] \cap [0, 2] = [0, 2]$.

After that, we follow computations from Homework 14 from last time.

Second system of equations. Based on the equations, we form the following four rules:

1. $x_1 \mapsto x_2 = -x_1$;
2. $x_2 \mapsto x_1 = -x_2$;
3. $x_1 \mapsto x_2 = -20/x_1$;
4. $x_2 \mapsto x_1 = -20/x_2$.

We start with $x_1 \in \mathbf{x}_1 = [0, 4]$ and $x_2 \in \mathbf{x}_2 = [-2, 0]$. We apply Rules 1, 2, 3, 4, etc. After the first two rules, we end up with the same situation as after the first two rules in Homework 14 from last time.