Solution to Problem 17

**Task.** Write down:

- an expression for which an optimizing compiler improves the estimates provided by straightforward interval computations, and
- an expression for which an optimizing compiler worsens improves the estimates provided by straightforward interval computations.

Your examples should be different from examples given in class (similar is OK).

**First example.** Let us consider the problem of estimating the expression $a \cdot b + a \cdot c$ when $a \in [-2, 2]$, $b = 1$ (i.e., $b \in [1, 1]$) and $c = -1$ (i.e., $c \in [-1, -1]$).

In this example, straightforward interval computations lead to

$$[-2, 2] \cdot [1, 1] + [-2, 2] \cdot [-1, -1] = [-2, 2] + [-2, 2] = [-4, 4].$$

For this example, optimizing compiler – aiming to minimize the number of multiplications – will transform the original expression into $a \cdot (b + c)$. For this new expression, straightforward interval computations lead to a narrower interval:

$$[-2, 2] \cdot ([1, 1] + [-1, -1]) = [-2, 2] \cdot [0, 0] = [0, 0].$$

**Second example.** Let us consider the problem of estimating the expression

$$\frac{1}{1 + \frac{a}{2b}}$$

when $a = b = [1, 2]$. In this case, straightforward interval computations lead to

$$\frac{1}{1 + \frac{[1, 2]}{2 \cdot [1, 2]}} = \frac{1}{1 + \frac{[1, 2]}{[2, 4]}} = \frac{1}{1 + [0.25, 1]} = \frac{1}{[1.25, 2]} = [0.5, 0.8].$$

For this example, optimizing compiler – aiming to minimize number of divisions – will transform the original expression into

$$\frac{2b}{2b + a}.$$
For this new expression, straightforward interval computations lead to a wider interval:

\[
\frac{2 \cdot [1, 2]}{2 \cdot [1, 2] + [1, 2]} = \frac{[2, 4]}{[2, 4] + [1, 2]} = \frac{[2, 4]}{[3, 6]} = \left[ \frac{1}{3}, \frac{4}{3} \right] = [0.33\ldots, 1.33\ldots].
\]