

Solution to Homework 1

Task. Use calculus to find the range of a function $f(x) = (4 + x) \cdot (2 - x)$ on the interval $[0, 6]$.

Solution. The derivative of $4 + x$ is 1, the derivative of $2 - x$ is -1 , so by using the formula for the derivative of the product, we conclude that

$$f'(x) = 1 \cdot (2 - x) + (4 + x) \cdot (-1) = 2 - x - 4 - x = -2 - 2x.$$

Thus, this derivative is equal to 0 when $-2 - 2x = 0$, i.e., when $x = -1$. This value is outside the range $[0, 6]$.

So, to find the range of the function, it is sufficient to compute its values at the endpoints $x = 0$ and $x = 6$.

- For $x = 0$, we have $f(0) = 4 \cdot 2 = 8$.
- For $x = 6$, we have $f(6) = 10 \cdot (-4) = -40$.

The smallest of these three values is -40 , the largest is 8, so the range is $[-40, 8]$.