Solution to Problem 24

Task. Let us assume that:

- the quantity x is described by the membership function

$$\mu(x) = \max(1 - |1 - x|, 0),$$

  and

- the quantity y is described by the membership function

$$\mu(y) = \max(1 - |-1 - y|, 0).$$

Use the values $\alpha = 0.25$, 0.5, 0.75, and 1.0 to describe membership functions for $z = x - y$ and $t = x \cdot y$.

Computing α-cuts for x.

- For $\alpha = 0.25$, the condition $\max(1 - |1 - x|, 0) \geq 0.25$ means that $1 - |1 - x| \geq 0.25$, so $|1 - x| = |x - 1| \leq 1 - 0.25 = 0.75$, i.e., $-0.75 \leq x - 1 \leq 0.75$. Adding 1 to all three sides of the inequality, we get $0.25 \leq x \leq 1.75$. Thus, the corresponding α-cut is $[0.25, 1.75]$.

- For $\alpha = 0.5$, the condition $1 - |1 - x| \geq 0.5$ means $|1 - x| = |x - 1| \leq 1 - 0.5 = 0.6$, i.e., $-0.5 \leq x - 1 \leq 0.25$. Adding 1 to all three sides of the inequality, we get $0.5 \leq x \leq 1.5$. Thus, the corresponding α-cut is $[0.5, 1.5]$.

- For $\alpha = 0.75$, the condition $1 - |1 - x| \geq 0.75$ means $|1 - x| = |x - 1| \leq 1 - 0.75 = 0.25$, i.e., $-0.25 \leq x - 1 \leq 0.25$. Adding 1 to all three sides of the inequality, we get $0.75 \leq x \leq 1.25$. Thus, the corresponding α-cut is $[0.75, 1.25]$.

- For $\alpha = 1$, the condition $1 - |1 - x| \geq 1$ means $|1 - x| = |x - 1| \leq 1 - 1 = 0$, i.e., $0 \leq x - 1 \leq 0$. Adding 1 to all three sides of the inequality, we get $1 \leq x \leq 1$. Thus, the corresponding α-cut is $[1, 1]$.

Computing α-cuts for y.

- For $\alpha = 0.25$, the condition $1 - |-1 - y| \ge 0.25$ means

$$|-1 - y| = |y + 1| \le 1 - 0.25 = 0.75,$$

i.e., $-0.75 \le y + 1 \le 0.75$. Subtracting 1 from all three sides of the inequality, we get $-1.75 \le y \le -0.25$. Thus, the corresponding α-cut is

$$[-1.75, -0.25].$$

- For $\alpha = 0.5$, the condition $1 - |-1 - y| \ge 0.5$ means

$$|-1 - y| = |y + 1| \le 1 - 0.5 = 0.5,$$

i.e., $-0.5 \le y+1 \le 0.5$. Subtracting 1 from all three sides of the inequality, we get $-1.5 \le y \le -0.5$. Thus, the corresponding α-cut is

$$[-1.5, -0.5].$$

- For $\alpha = 0.75$, the condition $1 - |-1 - y| \ge 0.75$ means

$$|-1 - y| = |y + 1| \le 1 - 0.75 = 0.25,$$

i.e., $-0.25 \le y + 1 \le 0.25$. Subtracting 1 from all three sides of the inequality, we get $-1.25 \le y \le -0.75$. Thus, the corresponding α-cut is

$$[-1.25, -0.75].$$

- For $\alpha = 1$, the condition $1 - |-1 - y| \ge 1$ means

$$|-1 - y| = |y + 1| \le 1 - 1 = 0,$$

i.e., $0 \le y + 1 \le 0$. Subtracting 1 from all three sides of the inequality, we get $-1 \le y \le -1$. Thus, the corresponding α-cut is

$$[-1, -1].$$

Computing α-cuts for $z = x - y$.

- For $\alpha = 0.25$, we have

$$[0.25, 1.75] - [-1.75, -0.25] = [0.25 - (-0.25), 1.75 - (-1.75)] = [0.5, 3.5].$$

- For $\alpha = 0.5$, we have

$$[0.5, 1.5] - [-1.5, -0.5] = [0.5 - (-0.5), 1.5 - (-1.5)] = [1, 3].$$

- For $\alpha = 0.75$, we have

$$[0.75, 1.25] - [-1.25, -0.75] = [0.75 - (-0.75), 1.25 - (-1.25)] = [1.5, 2.5].$$

- For $\alpha = 1$, we have

$$[1, 1] - [-1, -1] = [1 - (-1), 1 - (-1)] = [2, 2].$$

Computing α-cuts for $t = x \cdot y$.

- For $\alpha = 0.25$, we have

$$[0.25, 1.75] \cdot [-1.75, -0.25] =$$

$$[\min(0.25 \cdot (-1.75), 0.25 \cdot (-0.25), 1.75 \cdot (-1.75), 1.75 \cdot (-0.25)),$$

$$\max(0.25 \cdot (-1.75), 0.25 \cdot (-0.25), 1.75 \cdot (-1.75), 1.75 \cdot (-0.25))] =$$

$$[\min(-0.4375, -0.0625, -3.0625, -0.4375), \max(-0.4375, -0.0625, -3.0625, -0.4375)] =$$

$$[-3.0625, -0.0625].$$

- For $\alpha = 0.5$, we have

$$[0.5, 1.5] \cdot [-1.5, -0.5] =$$

$$[\min(0.5 \cdot (-1.5), 0.5 \cdot (-0.5), 1.5 \cdot (-1.5), 1.5 \cdot (-0.5)),$$

$$\max(0.5 \cdot (-1.5), 0.5 \cdot (-0.5), 1.5 \cdot (-1.5), 1.5 \cdot (-0.5)] =$$

$$[\min(-0.75, -0.25, -2.25, -0.75), \max(-0.75, -0.25, -2.25, -0.75)] =$$

$$[-2.25, -0.25].$$

- For $\alpha = 0.75$, we have

$$[0.75, 1.25] \cdot [-1.25, -0.75] =$$

$$[\min(0.75 \cdot (-1.25), 0.75 \cdot (-0.75), 1.25 \cdot (-1.25), 1.25 \cdot (-0.75)),$$

$$\max(0.75 \cdot (-1.25), 0.75 \cdot (-0.75), 1.25 \cdot (-1.25), 1.25 \cdot (-0.75))] =$$

$$[\min(-0.9375, -0.5625, -1.5625, -0.9375), \max(-0.9375, -0.5625, -1.5625, -0.9375)] = [-1.5625, -0$$

- For $\alpha = 1$, we have

$$[1, 1] \cdot [-1, -1] = [-1, -1].$$