Solution to Homework 25

Task. Suppose that we have three alternatives, for which the gains are in the intervals $[0, 100]$, $[30, 60]$, and $[80, 90]$.

- is there an alternative which is guaranteed to be optimal (i.e., for which the gain is the largest)?
- list all alternatives which can be optimal;
- which alternative(s) should we select if we use Hurwicz optimism-pessimism criterion with $\alpha = 0$, $\alpha = 0.5$, and $\alpha = 1$.

Solution.

- An alternative $[\underline{x}_i, \overline{x}_i]$ is guaranteed to be optimal if $\underline{x}_i \geq \overline{x}_j$ for all $j \neq i$. This is not true in this case:
$$\underline{x}_1 = 0 \not\geq \overline{x}_2 = 60,\ \underline{x}_2 = 30 \not\geq \overline{x}_1 = 100, \text{ and } \underline{x}_3 = 80 \not\geq \overline{x}_1 = 100.$$
- An alternative $[\underline{x}_i, \overline{x}_i]$ is possibly optimal if $\overline{x}_i \geq \max\limits_j \underline{x}_j$. In our case, $\max\limits_j \underline{x}_j = \max(0, 30, 80) = 80$. So, this inequality is satisfied for $i = 1$ and $i = 3$. So, the first and third alternatives are possibly optimal.
- In general, we compare the values $x_i = \alpha \cdot \overline{x}_i + (1 - \alpha) \cdot \underline{x}_i$. Here are the values for different α and i. For each α, the alternative with the largest value of x_i is underlined:

	$i = 1$	$i = 2$	$i = 3$
$\alpha = 0$	0	30	$\underline{80}$
$\alpha = 0.5$	50	45	$\underline{85}$
$\alpha = 1$	$\underline{100}$	60	90