Task. Use calculus to find the range of a function \( f(x_1, x_2) = (2 - x_1) \cdot (4 + x_2) \) when \( x_1 \) is in the interval \([0, 6]\) and \( x_2 \) is in the interval \([-1, 6]\).

Solution. The derivative with respect to \( x_1 \) is equal to \(-4 - x_2\), so this derivative is equal to 0 when \( x_2 = -4 \). The value \( x_2 = -4 \) is outside the interval \([-1, 6]\) for \( x_2 \), so we do not need to consider it. So, for \( x_1 \), we need to consider two cases:

- when \( x_1 = 0 \), and
- when \( x_1 = 6 \).

The derivative with respect to \( x_2 \) is equal to \( 2 - x_1 \), so this derivative is equal to 0 when \( x_1 = 2 \). This value is inside the interval \([0, 6]\) for \( x_1 \), so we need to consider this value. So, for \( x_2 \), we need to consider three cases:

- when \( x_2 = -1 \),
- when \( \frac{\partial f}{\partial x_2} = 0 \), i.e., when \( x_1 = 2 \), and
- when \( x_2 = 6 \).

First, let us consider cases when \( x_1 = 0 \).

- When \( x_1 = 0 \) and \( x_2 = -1 \), we get \( f(x_1, x_2) = (2+0) \cdot (4+(-1)) = 2 \cdot 3 = 6 \).
- The case when \( x_1 = 0 \) and \( x_1 = 2 \) is not possible since \( 0 \neq 2 \).
- When \( x_1 = 0 \) and \( x_2 = 6 \), we get \( f(x_1, x_2) = 2 \cdot 10 = 20 \).

Second, we consider cases when \( x_1 = 6 \):

- When \( x_1 = 6 \) and \( x_2 = -1 \), we get \( f(x_1, x_2) = (-4) \cdot 3 = -12 \).
- The case when \( x_1 = 6 \) and \( x_1 = 2 \) is not possible since \( 6 \neq 2 \).
- When \( x_1 = 6 \) and \( x_2 = 6 \), we get \( f(x_1, x_2) = (-4) \cdot 10 = -40 \).

So, we get the following values:
\[ \frac{\partial f}{\partial x_2} = 0, \text{ i.e., } x_1 = 2 \]

<table>
<thead>
<tr>
<th>( x_2 = -1 )</th>
<th>( x_1 = 0 )</th>
<th>( x_1 = 6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>-12</td>
<td></td>
</tr>
</tbody>
</table>


\[ \frac{\partial f}{\partial x_2} = 0, \text{ i.e., } x_1 = 2 \]

<table>
<thead>
<tr>
<th>( x_2 = 6 )</th>
<th>( x_1 = 0 )</th>
<th>( x_1 = 6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>N/A</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>-40</td>
<td></td>
</tr>
</tbody>
</table>

The smallest of these values is \(-40\), the largest is 20, so the range is \([-40, 20]\).