Solution to Homework 2

Task. Use calculus to find the range of a function $f(x_1, x_2) = (2 - x_1) \cdot (4 + x_2)$ when x_1 is in the interval $[0, 6]$ and x_2 is in the interval $[-1, 6]$.

Solution. The derivative with respect to x_1 is equal to $-4 - x_2$, so this derivative is equal to 0 when $x_2 = -4$. The value $x_2 = -4$ is outside the interval $[-1, 6]$ for x_2, so we do not need to consider it. So, for x_1, we need to consider two cases:

- when $x_1 = 0$, and
- when $x_1 = 6$.

The derivative with respect to x_2 is equal to $2 - x_1$, so this derivative is equal to 0 when $x_1 = 2$. This value is inside the interval $[0, 6]$ for x_1, so we need to consider this value. So, for x_2, we need to consider three cases:

- when $x_2 = -1$,
- when $\dfrac{\partial f}{\partial x_2} = 0$, i.e., when $x_1 = 2$, and
- when $x_2 = 6$.

First, let us consider cases when $x_1 = 0$.

- When $x_1 = 0$ and $x_2 = -1$, we get $f(x_1, x_2) = (2+0) \cdot (4+(-1)) = 2 \cdot 3 = 6$.
- The case when $x_1 = 0$ and $x_1 = 2$ is not possible since $0 \neq 2$.
- When $x_1 = 0$ and $x_2 = 6$, we get $f(x_1, x_2) = 2 \cdot 10 = 20$.

Second, we consider cases when $x_1 = 6$:

- When $x_1 = 6$ and $x_2 = -1$, we get $f(x_1, x_2) = (-4) \cdot 3 = -12$.
- The case when $x_1 = 6$ and $x_1 = 2$ is not possible since $6 \neq 2$.
- When $x_1 = 6$ and $x_2 = 6$, we get $f(x_1, x_2) = (-4) \cdot 10 = -40$.

So, we get the following values:

	$x_1 = 0$	$x_1 = 6$
$x_2 = -1$	6	-12
$\dfrac{\partial f}{\partial x_2} = 0$, i.e., $x_1 = 2$	N/A	N/A
$x_2 = 6$	20	-40

The smallest of these values is -40, the largest is 20, so the range is $[-40, 20]$.