Solution to Homework 3

Task. Linearize the expression from Problem 1 around the interval's midpoint. Use the linearized expression to find the approximate value of the range of the original function, both with the actual derivative and with the result of numerical differentiation.

Solution: preliminary computations. The midpoint of the interval $[0, 6]$ is the point $\widetilde{x}_1 = \dfrac{0+6}{2} = 3$. For this value, we get

$$\widetilde{y} = f(\widetilde{x}_1) = (4 + \widetilde{x}_1) \cdot (2 - \widetilde{x}_1) = (4+3) \cdot (2-3) = 7 \cdot (-1) = -7.$$

The half-width of the interval is equal to $\Delta_1 = \dfrac{6-0}{2} = 3$.

Case when we consider the actual derivative. Here,

$$f'(\widetilde{x}_1) = -2 - 2\widetilde{x}_1 = -2 - 2 \cdot 3 = -2 - 6 = -8.$$

Thus,

$$\Delta = |f'(\widetilde{x}_1)| \cdot \Delta_1 = |-8| \cdot 3 = 8 \cdot 3 = 24.$$

So, the estimated range is

$$[\widetilde{y} - \Delta, \widetilde{y} + \Delta] = [-7 - 24, -7 + 24] = [-31, 17],$$

not a very good approximation, by the way.

Case when we consider numerical differentiation. In this case,

$$\Delta = |f(\widetilde{x}_1 + \Delta_1) - f(\widetilde{x}_1)|.$$

We have found that $f(\widetilde{x}_1) = -7$. From the solution of Problem 1, we know that

$$f(\widetilde{x}_1 + \Delta_1) = f(\overline{x}_1) = f(6) = -40.$$

So, $\Delta = |-7 - (-40)| = 33$, and thus, the estimated range is

$$[\widetilde{y} - \Delta, \widetilde{y} + \Delta] = [-7 - 33, -7 + 33] = [-40, 26].$$

In this case, at least the lower endpoint is correct.