# Solution to Homework 4

**Task.** Linearize the expression from Problem 2 around the intervals' midpoints. Use the linearized expression to find the approximate value of the range of the original function, both with the actual derivative and with the result of numerical differentiation.

**Solution: preliminary computations.** The midpoint of the interval $[0, 6]$ is the point $\widetilde{x}_1 = \dfrac{0+6}{2} = 3$, the midpoint of the interval $[-1, 6]$ is

$$\widetilde{x}_2 = \frac{(-1)+6}{2} = 2.5.$$

For these values, we get

$$\widetilde{y} = f(\widetilde{x}_1, \widetilde{x}_2) = (2 - \widetilde{x}_1) \cdot (4 + \widetilde{x}_2) = (2 - 3) \cdot (4 + 2.5) = (-1) \cdot 6.5 = -6.5.$$

The half-width of the first interval is equal to $\Delta_1 = \dfrac{6-0}{2} = 3$, the half-width of the second interval is equal to $\Delta_2 = \dfrac{6-(-1)}{2} = 3.5$.

**Case when we consider the actual derivatives.** Here,

$$c_1 = \frac{\partial f}{\partial x_1}(\widetilde{x}_1, \widetilde{x}_2) = -4 - \widetilde{x}_2 = -4 - 2.5 = -6.5,$$

and

$$c_2 = \frac{\partial f}{\partial x_2}(\widetilde{x}_1, \widetilde{x}_2) = 2 - \widetilde{x}_1 = 2 - 3 = -1.$$

Thus,

$$\Delta = |c_1| \cdot \Delta_1 + |c_2| \cdot \Delta_2 = 6.5 \cdot 3 + 1 \cdot 3.5 = 19.5 + 3.5 = 23.$$

Thus, the estimated range is

$$[\widetilde{y} - \Delta, \widetilde{y} + \Delta] = [-6.5 - 23, -6.5 + 23] = [-29.5, 16.5].$$

**Case when we consider numerical differentiation.** In this case,

$$\Delta = |f(\widetilde{x}_1 + \Delta_1, \widetilde{x}_2) - \widetilde{y}| + |f(\widetilde{x}_1, \widetilde{x}_2 + \Delta_2) - \widetilde{y}| =$$

$$|f(6, 2.5) - (-6.5)| + |f(3, 6) - (-6.5)|.$$

Here,

$$f(6, 2.5) = (2 - 6) \cdot (4 + 2.5) = (-4) \cdot 6.5 = -26$$

and

$$f(3, 6) = (2 - 3) \cdot (4 + 6) = (-1) \cdot 10 = -10.$$

Thus,

$$\Delta = | - 26 + 6.5| + | - 10 + 6.5| = 19.5 + 3.5 = 23.$$

So, the estimated range is

$$[\widetilde{y} - \Delta, \widetilde{y} + \Delta] = [-6.5 - 23, -6.5 + 23] = [-29.5, 16.5].$$