

Test 1 for Interval Class, Fall 2023

Problem 1. Why do we need data processing?

Problem 2. Why do we need interval computations?

Problem 3. What is the main problem of interval computations?

Problem 4. Use calculus to find the range of a function $f(x) = (x-1) \cdot (2-x)$ on the interval $[1.2, 1.6]$. Explain the main algorithm of using calculus to find the range.

Problem 5. Use calculus to find the range of a function

$$f(x_1, x_2) = (x_1 - 1) \cdot (2 - x_2)$$

when x_1 is in the interval $[0.8, 1.2]$ and x_2 is in the interval $[-1.2, -0.8]$. Why cannot we use calculus to find the range for any number of inputs?

Problem 6–7. Linearize the expression from Problem 4 around the interval's midpoint. Use the linearized expression to find the approximate value of the range of the original function, both with the actual derivative and with the result of numerical differentiation. Write down the general formulas for both linearization methods; in each formula, explain what each variable means.

Problem 8–9. Linearize the expression from Problem 5 around the intervals' midpoints. Use the linearized expression to find the approximate value of the range of the original function, both with the actual derivative and with the result of numerical differentiation.

Problem 10. Use bisection to find $\sqrt{5}$ – i.e., the solution to the equation $x^2 - 5 = 0$ – with accuracy $\varepsilon = 0.25$. Start with the interval $[0, 4]$.