Test 3 for Interval Class, Fall 2023

Problem 1. Use the interval-based optimization algorithm to locate the maximum of the function $x^2 + x + 1$ on the interval $[-0.6, -0.2]$. Divide this interval into two, then divide the remaining intervals into two again, etc. At each iteration, dismiss subintervals where maximum cannot be attained. Stop when you get intervals of width 0.1.

Problem 2–3. Use the constraints method to solve the following two problems:

- find x_1 from the interval $[0, 1]$ and x_2 from the interval $[0, 1]$, for which $x_1 + x_2 = 1$ and $x_1 \cdot x_2 = 0.25$;
- find x_1 and x_2 from the same intervals, for which $x_1 + x_2 = 1$ and $x_1 \cdot x_2 = 0.5$.

For each problem, perform two cycles, in each of which you apply all the rules one by one.

Problem 4. For the function $x_1 \cdot x_2 + 0.5 \cdot x_1 + 0.5 \cdot x_2$, when $\widetilde{x}_1 = -0.4$ and $\widetilde{x}_2 = -0.6$, and the standard deviations are $\sigma_1 = \sigma_2 = 0.1$, estimate the variance σ^2 of the result of data processing.

Problem 5. Write down:

- an expression for which an optimizing compiler improves the estimates provided by straightforward interval computations, and
- an expression for which an optimizing compiler worsens improves the estimates provided by straightforward interval computations.

Your examples should be different from examples given in class (similar is OK).

Problem 6. If we only use numbers with one digit after the decimal point, and we use rounding that preserves guaranteed bounds, what will be the result of multiplying the intervals $[0.5, 0.7]$ and $[0.3, 0.6]$?

Problem 7. Two questions:

- If out of 20 experts, 16 think that the statement is true, what degree of confidence shall we assign to this statement?
- If an expert marked her confidence in a statement as 4.5 on a scale from 0 to 5, what degree of confidence shall we assign to this statement?

Problem 8. If we have $\mu(0) = 1$ and $\mu(5) = 0$, what value shall we assign to $\mu(2)$? Use linear interpolation.

Problem 9. If the expert's degrees of confidence in two statements A and B are 0.7 and 0.9, and we use min as an "and"-operation and max as an "or"-operation, what degree of confidence shall we assign to $A \,\&\, B$? To $A \vee B$? If a statement C has degree of confidence 0.8, what is our degree of belief in $(A \,\&\, B) \vee C$?

Problem 10. Suppose that we have three alternatives, for which the gains are in the intervals $[0, 50]$, $[20, 30]$, and $[30, 40]$.

- is there an alternative which is guaranteed to be optimal (i.e., for which the gain is the largest)?
- list all alternatives which can be optimal;
- which alternative(s) should we select if we use Hurwicz optimism-pessimism criterion with $\alpha = 0$, 0.5, and 1.