CS 4390/5353, Quantum Computing, Test 1
September 16, 2004

Name: ....

Conditions: No books allowed; you can use 5 pages of handwritten notes.

1. What are the probabilities of 0 and 1 in the state

$$\frac{1+2\cdot \mathrm{i}}{\sqrt{13}}|0\rangle + \frac{\sqrt{12}}{\sqrt{13}}|1\rangle$$

Check whether this state is physically valid.

2. Explain, on an example, why we need reversibility in quantum mechanics.

Hint: otherwise, we may end up with a state that is not physically valid, because the probabilities will not add up to 1.

3. Describe the conjunction ("and") operation & as a Toffoli gate; write down the truth table for this gate.

4. Deutsch-Josza algorithm: general

   4a. Describe what is the problem solved by the Deutsch-Josza algorithm, i.e., what is the input and what is the desired output.

   4b. List the steps of this algorithm.

   4c. Explain in what sense this algorithm is better than non-quantum algorithms for solving the same problem.

5–7. Show, step-by-step, that for the function $f(x) \equiv 0$, the Deutsch-Josza algorithm leads to the correct answer.

8–9. Entanglement.

8a. Explain how Controlled Not (Toffoli gate corresponding to the identity function $f(x) = x$) can be used for copying classical states.

8b. Show that for quantum states, this same gate leads to entanglement.

9a. Explain what is entanglement.

9b. Give an example of an entangled state, and explain the Einstein-Podolsky-Rosen (EPR) paradox on this example.

10. Grover's algorithm.

10a. Describe what is the problem solved by the Grover's algorithm, i.e., what is the input and what is the desired output.

10b. Explain in what sense this algorithm is better than non-quantum algorithms for solving the same problem.

10c. *For extra credit:* describe the main steps of Grover's algorithm.