We had data fusion (ideal) 9/9/8

We have several measurments of same quantity.
(maybe with different / same accuracy)

Same

$\sigma_1, \ldots, \sigma_n \qquad \sigma_1 = \ldots = \sigma_n$

Weighted Avg.

$$\sigma^2 = \frac{1}{\frac{1}{\sigma_1^2} + \ldots + \frac{1}{\sigma_n^2}}$$

$$\sigma^2 = \frac{\sigma_1^{\,3}}{n}$$

$$\sigma = \frac{\sigma_1}{\sigma_n}$$

rule of thumb
accuracy of
stat est. based
on sample size
n is $\sim \frac{1}{\sqrt{n}}$

- Data Fusion (ideal) if we have the quantity, we can directly measure.

4:52 ⚠ But many cases we cannot measure everything.
In many situations: — many quantities can not be easily measured.

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e.g., any quantities inside the Earth density

t°, ...

- distance for a star

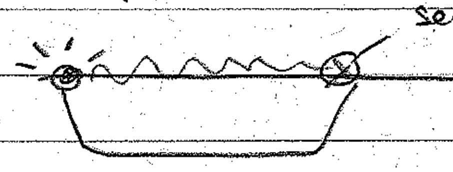
there are 2 way to meaure density:

- Seismic experiments.

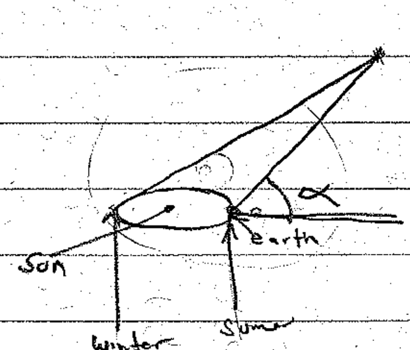
16:04 Depending on density you can have diff directions

→ active exp.

make an explosion!



How do you measure distance of a star.



→ passive exp e.g., the observatory

We find some quantities $x_1, \dots, x_n$ which are easier to measure and which are related to y: $y = f(x_1, \dots, x_n)$

x_1 - mass

x_2 - volume

$$y = \frac{x_1}{x_2} \quad f(x_1, x_2) = \frac{x_1}{x_2}$$

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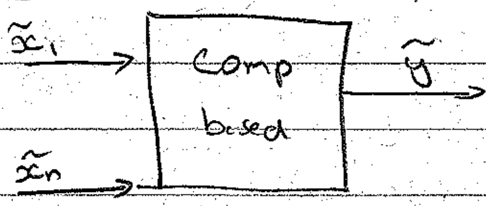
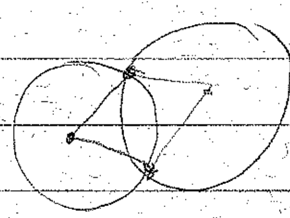
Indirect measurement

- We measure auxiliary quantities $\tilde{x}_1, \dots, \tilde{x}_n$
- We plug into f: $\tilde{y} = f(\tilde{x}_1, \dots, \tilde{x}_n)$

27.00 $\boxed{\text{Additional Problem: } f \text{ is not known exactly}}$

What we have:





How determine where is the enemy?

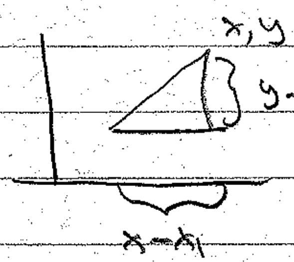
2 - radar

How find the coordinates,

$X_1 \quad Y_1$ ← 1st radar

$X_2 \quad Y_2$ ← 2nd radar

$$(x_1 - x)^2 + (y_1 - y)^2 = d_1^2$$
$$(x_2 - x)^2 + (y_2 - y)^2 = d_2^2$$



△ If we plug-in

$$(y_1 - y)^2 = d_1^2 - (x_1 - x)^2$$
$$y_1 - y = \sqrt{d_1^2 - (x_1 - x)^2}$$
$$y = y_1 - \sqrt{d_1^2 - (x_1 - x)^2}$$

Substitute

Or by using numeric methods s.a. newton, This is how radars work!!

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$$x^2 - 2x_1 x + x_1^2 + y^2 - 2y_1 y + y_1^2 = d_1^2$$
$$x^2 - 2x_2 x + x_2^2 + y^2 - 2y_2 y + y_2^2 = d_2^2$$

we subtract

$$-2(x_1 - x_2)x + x_1^2 - x_2^2 -$$
$$2(y - y_2)y + y_1^2 - y_2^2 = d_1^2 - d_2^2$$

$$2(y_1 - y_2)y = d_1^2 - d_2^2 - 2(x_1 - x_2)x + x_1^2 - x_2^2 + y_1^2 - y_2^2$$

40:17

$$y = Ax + B \qquad A = -\frac{x_1 - x_2}{y_1 - y_2} \qquad B = \frac{d_2^2 + d_1^2 + x_1^2 - x_2^2 + y_1^2 - y_2^2}{2(y_1 - y_2)}$$

We have a value that is obtained from the results of the direct measurements

we have : $\tilde{y} = f(\tilde{x}_1, \ldots, \tilde{x}_n)$

we want : $y = f(x_1, \ldots, x_n)$

Measurements are never absolutely accurate $\tilde{x}_1 \neq x_1$

$$\Delta x_2 \overset{\text{def}}{=} \tilde{x}_1 - x_2 \neq 0$$

Some density of a crown is

$y = 6.75$,

actual value

we have the actual value or value.

$y = 6.75 \qquad x_2 = 1.1 \qquad \tilde{x}_2 = 1 \qquad \tilde{y} = \frac{7}{1} = 7$

$x_1 = 7.425 \qquad \tilde{x}_1 = 7$

$\boxed{Q}$ How accurate is this result?

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49:22 **Question:** How do we gauge the diff.?

$$\Delta y = \tilde{y} - y?$$

What is the guarantee? how do we trust it?

How do we find the difference?

$$X_i = \tilde{x}_i - \Delta x$$

X_i: unknown

$\tilde{x}_i$: known

Δx: unknown

normally distributed with

0 mean

unknown std. deviation

σ_i

$$\Delta y = \tilde{y} - y =$$

measured value $= f(\tilde{x}_1, \ldots, \tilde{x}_n) - f(x_1, \ldots, x_n) =$

$$f(\tilde{x}_1, \ldots, \tilde{x}_n) - f(\tilde{x}_1 - \Delta x_1, \ldots, \tilde{x}_n - \Delta x_n)$$

! Good news.

Δx_i are small

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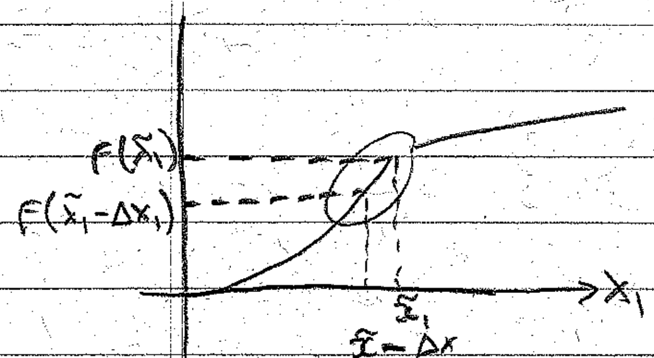
Example of 1 variable

$F(\tilde{x}_1) - f(\tilde{x}_1 - \Delta x_1)$



$$\frac{\Delta y}{\Delta x_1} \approx \frac{\partial f}{\partial x_1}$$

$$\boxed{\Delta y \approx \frac{\partial f}{\partial x} \Delta x_1}$$