Prob Interval Unc

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- Possible eq:

1) Given a function f
   Measured Values $\tilde{x}_1, \ldots, \tilde{x}_n$
   " error $\boxed{\Delta x_1}, \ldots \Delta x_n$

$x_1, \ldots, x_n$

Δx_n

Compute
→ actual value, measured error
→ linearization.

The result of Data processing

$$f = x_1 \cdot x_2 + 3x_1 - x_2$$

$\tilde{x}_1 = 1.0$ $\quad$ $\tilde{x}_2 = 3.0$

$\Delta x_1 = -0.1$ $\quad$ $\Delta x_2 = +0.1$

$$\tilde{y} = 1 \cdot 3 + 3(1) - 3$$
$$\boxed{\tilde{y} = 3.0}$$

two diff ways.
① Plug-in ② linearization

$$\Delta x = \tilde{x} - x$$

$x_1 = \tilde{x}_1 - \Delta x_1 = 1.1$

$x_2 = \tilde{x}_2 - \Delta x_2 = 2.9$ →

$$y = 1.1 \cdot 2.9 + 3 \cdot 1.1 - 2.9$$
$$\boxed{y = 3.59}$$

$$\boxed{\Delta y = -0.59}$$

$$\Delta y = \frac{\partial f}{\partial x_1} \cdot \Delta x_1 + \frac{\partial f}{\partial x_2} \cdot \Delta x_2$$

we get → $\Delta y = 6 \cdot (-0.1) + 0 \cdot (0.1)$

$$\boxed{\Delta y = -0.6}$$

$\left.\begin{array}{l} \frac{\partial f}{\partial x_1} = x_2 + 3 \Rightarrow 3 + 3 = 6 \\ \frac{\partial f}{\partial x_2} = x_1 - 1 = 1 - 1 = 0 \end{array}\right\}$ by plugging here

We compare

They are close!

2) Data Fusion:

given values (same quantity): $\left.\begin{array}{l}\tilde{x}_1, \ldots, \tilde{x}_n \\ \sigma_1, \ldots, \sigma_n\end{array}\right\}$ given.

we want: $\tilde{x}$, σ

$$\tilde{x} = \frac{\tilde{x}_1 \cdot \tilde{\sigma}_1^{-2} + \ldots + x_n \sigma_n^{-2}}{\sigma_1^{-2} + \ldots + \sigma_n^{-2}}$$

$$\sigma^2 = \frac{1}{\sigma_1^{-2} + \ldots + \sigma_n^{-2}}$$

e.g.

Measurement results

$\tilde{x}_1 = 1.1$, $\sigma_1 = 0.1$

$\tilde{x}_2 = 0.9$, $\sigma_2 = 0.3$

$$\tilde{x} = \frac{(1.1 * \cancel{100})^{OK} + (0.9 * \cancel{\frac{100}{9}})^{OK}}{\cancel{100}^{OK} + \cancel{\frac{100}{9}}^{OK}} = \frac{110 + 10}{\frac{900 + 100}{9}} = \frac{120 \cdot 9}{1000} = \frac{1080}{1000}$$

$$\boxed{\tilde{x} = 1.08}$$

$$\sigma^2 = \frac{1}{100 + \frac{100}{9}} = \frac{9}{1000} \quad \boxed{\sigma = \frac{3}{10 \cdot \sqrt{10}}}$$

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3) <u>Given</u> : $f_1, \tilde{x}_1, \dots, \tilde{x}_n, \sigma_1, \dots, \sigma_n$

we want $\sigma = \sigma[\Delta y]$ → <u>analytical diff</u>

→ <u>Numerical diff.</u>

$$f = x_1 \cdot x_2 + 3x_1 - x_2$$

$\tilde{x}_1 = 1.0 \qquad \tilde{x}_2 = 3.0$

$\sigma_1 = 0.3 \qquad \sigma_2 = 0.1$

<u>Analytic</u>

$$\sigma = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i}\right)^2 \sigma_i^2} \qquad \sigma^2 = 6^2 \cdot (0.3)^2 + 0^2 \cdot (0.1)^2 = 6^2 \cdot 0.3^2$$

$$\frac{\partial f}{\partial x_1} = 6 \qquad \frac{\partial f}{\partial x_2} = 0 \qquad \sigma = 6 \cdot 0.3 = 1.8$$

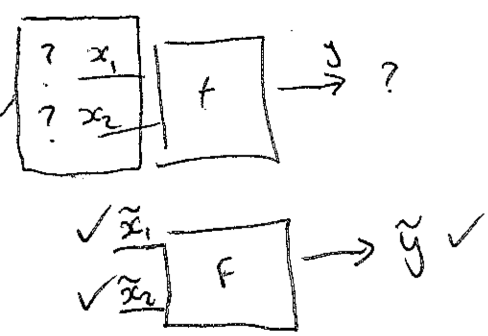
$$\boxed{\sigma = 1.8}$$

<u>Numeric. diff..</u>

$$\sigma^2 = \Big(\underbrace{f(x_1+\sigma_1, x_n)}_{(1.3)\cdot 3 + 3(1.3) - 3 \,=\, 4.8} - \underbrace{\tilde{y}}_{=\,3.0}\Big)^2 + \Big(\underbrace{f(x_1, x_2+\sigma_2)}_{1.0\cdot 3 + 3\cdot(1.0) - 3.1 \,=\, 3.0} - \tilde{y}\Big)^2$$

$$(1.8)^2 + 0^2$$

$$\boxed{\sigma = 1.8}$$



we don't know the actual accuracy. Δx

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Computational Questions:

1) When Monte Carlo faster? $\longrightarrow N \quad \frac{1}{\sqrt{N}}$

- provide algorithms.
- explain each steps.

When Num. diff. faster? $n+1$ $\quad \sigma^2 = \sum \left(f(\tilde{x}_1, \dots, \tilde{x}_{n-1}, \tilde{x}_i + \sigma_i, \tilde{x}_{i+1}, \dots) - \tilde{y} \right)^2$

(2) we start with Formula Part. diff.

we end up: $\quad \sigma^2 = \sum \left(\frac{\left(f(\tilde{x}_1, \dots, \tilde{x}_{i-1}, \tilde{x}_i + h_i, \dots, \tilde{x}_n \right) - \tilde{y}}{h_i} \right)^2 \sigma_i^2$

One case when $h_i = \sigma_i$

we cut up calculations

Motivations

What is Cyber-inf.? Why we need it? why we need to estimate

instead of moving data around we keep it

Why data processing?

Some things are hard to measure.

- How simulate Gauss distribution / Monte carlo sim.

Mathematical Questions

$x_1, \dots, x_n$ come from a & σ we use the Maximum likelyhood method

We omit Δx 'cos doesn't depend on a & σ

the prob $\frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(-\frac{(x_1 - a)^2}{2\sigma^2}\right) \Delta x \cdots \frac{1}{\sqrt{2\pi}\,\sigma} \cdot \exp\left(\frac{-(x_n-a)^2}{\sigma^2}\right) \Delta x \rightarrow \underset{a,\sigma}{\mathrm{Max}}$