Tibor

10/14/8

CP

Modeling

↓

Solving → (1) Filtering / Pruning

↓ → (2) Branching

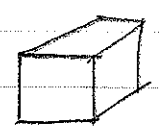
Solutions

Quasi-solutions :-

⚠ When is critical we use I.C.

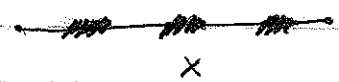
The constraints are expressed in terms of C, V, D

You end-up with a box of possible



The solver will repeat these two processes (1) & (2) to have more refined solutions,

Domain of x :

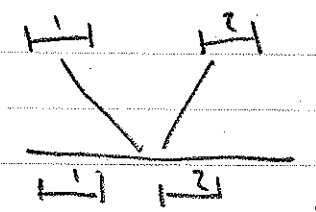


→ Cannot be a solution

Filtering

Filters from the domain the values that cannot be a solution

For continuous domains: Are based on Intervals



← Queue that holds intervals to process later.

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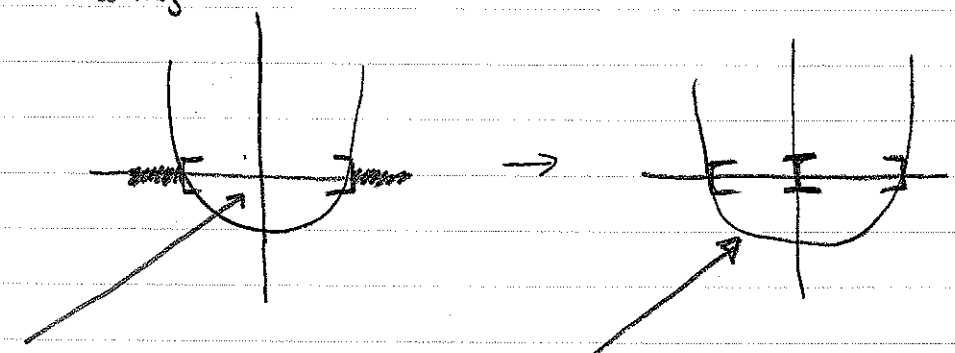
The branching:



You split → You have two intervals, to make sure we don't miss any solution.

Filtering → discrete domains
Pruning → continuous domains

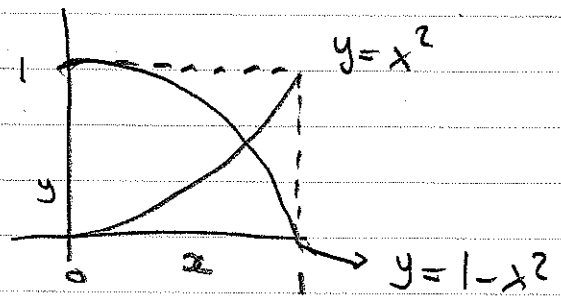
<u>How achieve Filtering & Pruning?</u>

We achieve it through what is called local consistency techniques.

- Consistency :- how likely is satisfied the constraints.
  a domain is inconsistent we do not need it.
  consistent ⇝ we don't know yet until we know better.

- Local Consistency technique:
  constraints : $\{C_1, C_2, \ldots, C_m\}$
  we process $C_1$ with $D_1$,
  $C_2$ with $D_2$..
  ⋮

$$\boxed{\begin{array}{l} X = \{x, y\} \\ C = \{C_1: y = x^2 \\ \quad\quad C_2: y = 1 - x^2\} \\ D = [0,1] \times [0,1] \end{array}}$$




$C_1$: $x = [0,1]$
$y = [0,1]$
all elements from $[0,1]^2$ are solutions.

$C_2$: $x = [0,1]$
$y = [0,1]$

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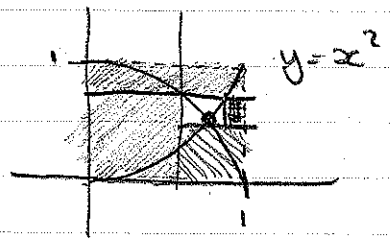
Branching :- We help the solving process a lot.
⚠ There is a lot of heuristics about how branch.




two boxes that generate worse generate duplicates.

<u>local consistency</u>: We consider one constraint at the time. and doesn't propagate results.

<u>Global Consistency</u>: Take the package of constraints. You apply linear Programming in these Constraints.
e.g. of global constraints. "<u>all diff</u>"
all

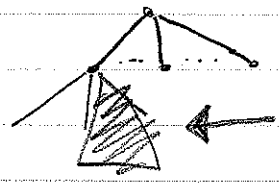
X Drawback :- In order to use global constraints, has to fit your constraints in Global

Time-Complexity is a NP-Problem.

In theoretical is hard to solve it, but practical (using heuristics) it works very well.

<u>Propagation</u>: - everytime you make a decision, you propagate that decision to other decisions when consider this...

↑ The basis of solvers.



← eliminate the possible tree because we know the decision

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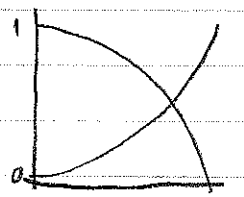
CP: has a lot of potential to solve problems.
efficient enough: - because it is used...

Sometimes is not recommended to use CP solvers (e.g. Cless)
but is recommended to use feeders from the CP solvers.

When you don't know your constraints you use
'speculative constraints'

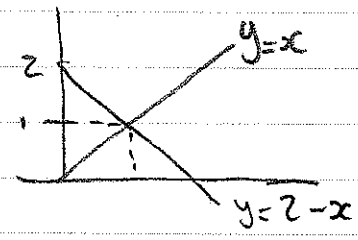
☑ You don't have to start from scratch.
meta-solving stage.




$C_1: \; y = x^2 = [0,1]^2$
$= [0,1] \cap [0,1] = [0,1]$

$x = \sqrt{y} = \sqrt{[0,1]} = [-1,1] \cap [0,1] = [0,1]$

$C_1: \; y = x^2 = [0, \frac{1}{2}] = [0, \frac{1}{4}] \cap [0,1] = [0, \frac{1}{4}]$
$x = \sqrt{y} = \sqrt{[0, \frac{1}{4}]} = [\pm \frac{1}{2}] \cap [0, \frac{1}{2}] = [0, \frac{1}{2}]$




$C_1 = x, y$
$C_2 = y = 2 - x = 2 - [0,1]$
$= [1,2] \cap [0,1] = \{1\}$
$x = 2 - y = 2 - \{1\} = \{1\}$

$C_1: x, y$

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<u>So far:</u>

$x_1, \dots, x_n$,

$y = f(x_1, \dots, x_n)$ ← Known relation

$\tilde{x}_1, \dots, \tilde{x}_n$ ← measurment results.

$\tilde{y} = f(\tilde{x}_1, \dots, \tilde{x}_n)$ - results of data processing

We know $\Delta x_i = \tilde{x}_i - x_i \neq 0$

<u>So far.</u> assuming that $\Delta x_1, \dots, \Delta x_n$ are normally distr, w/o mean and known standard deviation $\sigma_i$

$\Delta y \to$ normally distr, w/ 0 mean and st. dev

$$\sigma = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial f}{\partial x}\right)^2 \sigma_i^2}$$

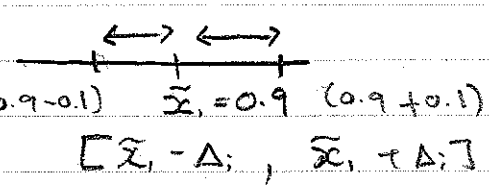
<u>In practice</u> often we actually don't know the distribution. All we know is the upper bound on the measurment error: $\Delta_1, \dots, \Delta_n$: $|\Delta x_i| \leq \Delta_i$

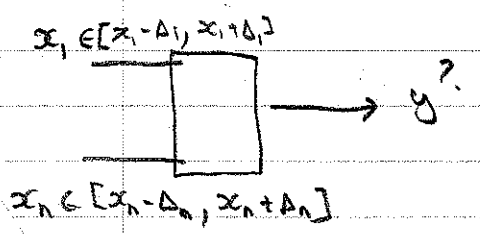
$x_i \in [\tilde{x}_i - \Delta_i, \tilde{x}_i + \Delta_i]$

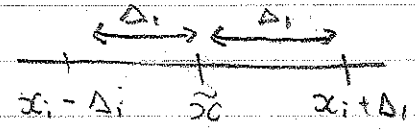
<u>Question:</u> What do we know about $\Delta y$?

$\tilde{x}_1 = 0.9$

$\Delta_1 = 0.1$



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Ohms law:

$V = I R$

$\tilde{I} = 1.0 \quad \Delta_I = 0.1 \quad I \in [0.9, 1.1]$

$\tilde{R} = 2.0 \quad \Delta_R = 0.2 \quad R \in [1.8, 2.2]$

$V = 1 \cdot 2 = 2.0$

$V = 0.9 \cdot 2.2 = 1.98$

$V = 1.1 \cdot 1.8 = 1.98$

$V = 1.05 \cdot 2.0 = 2.1$

The voltage can be between.

$[V] = [0.9 \cdot 1.8, \ 1.1 \cdot 2.2]$

$= [1.62, 2.42]$

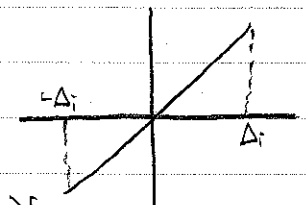
The actual value may differ

In the linear case:

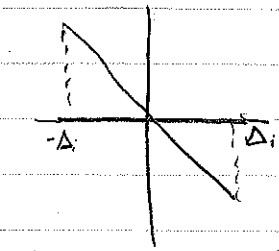
$$\Delta y = \sum_{i=1}^{n} \frac{\partial f}{\partial x_i} \Delta x_i$$

this is fixed

$\Delta x_i \in [-\Delta_i, \Delta_i]$




$\frac{\partial f}{\partial x_i} > 0 \Rightarrow \frac{\partial f}{\partial x_i} \cdot \Delta_i$




$\frac{\partial f}{\partial x_i} < 0 \Rightarrow \frac{\partial f}{\partial x_i} \cdot (-\Delta_i)$

this is expressed as:

$\left(-\frac{\partial f}{\partial x_i}\right) \cdot \Delta_i$

When is $\Delta y$ the largest?

Inputs: $\Delta x_1, \ldots, \Delta x_n$.

property: $\Delta y$, is monotonic in each $\Delta x_i$.

$$\boxed{\Delta = \sum_{i=1}^{n} \left| \frac{\partial f}{\partial x_i} \right| \Delta_i}$$

(absolute value)

Generic.

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Current

$\tilde{I} = 1.0 \qquad \Delta_I = 0.1$

$\tilde{R} = 2.0 \qquad \Delta_R = 0.2$

$F(I, R) = I \cdot R$

$\tilde{x}_1 = 1.0 \qquad \Delta_1 = 0.1 \qquad \dfrac{\partial f}{\partial x_1} = x_2 = 2.0$

$\tilde{x}_2 = 2.0 \qquad \Delta_2 = 0.2$

$F(x_1, x_2) = x_1 \cdot x_2 \qquad \dfrac{\partial f}{\partial x_2} = x_1 = 1.0$

$\tilde{y} = 2$

$[1.62, 2.42]$ $\qquad \Delta = |2.0| \cdot 0.1 + |1.0| \cdot 0.2 = \underline{0.4}$

actual range

They are close

$y \in [\tilde{y} - \Delta, \tilde{y} + \Delta]$

$[2.0, 0.4]$

$[1.6, 2.4]$

Homework

1) run a numerical example of interval computation. compare with linearized case.

$a+b$

$a+b^2$

Numerical differentiation

$$\frac{\partial f}{\partial x_i} = \frac{f(\tilde{x}_1, \dots, \tilde{x}_{i-1}, \tilde{x}_i + h_i, \tilde{x}_{i+1}, \dots, \tilde{x}_n) - \tilde{y}}{h_i}$$

$$\Delta = \sum_{i=1}^{n} \left| \frac{f(\tilde{x}_1, \dots, \tilde{x}_{i-1}, \tilde{x}_i + h_i, \tilde{x}_{i+1}, \dots, \tilde{x}_n) - \tilde{y}}{h_i} \right| \cdot \Delta_i$$

To simplify computations we choose $h_i = \Delta_i$

$$\Delta = \sum_{i=1}^{n} \left| f(\tilde{x}_1, \dots, \tilde{x}_{i-1}, \tilde{x}_i + \Delta_i, \tilde{x}_{i+1}, \dots, \tilde{x}_n) - \tilde{y} \right|$$

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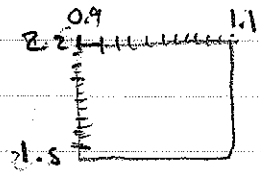
Homework.

2) Write a code to compute $\Delta$ by using numerical diff & check by exhaustive search

How can we check it is correct




$x_1 \in [x_1 - \Delta_1, \tilde{x}_1 + \Delta_1]$

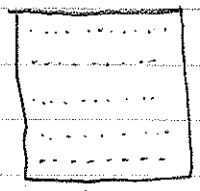
```
For ( x1 = x̃i - Δ1, x1 <= x̃i + Δ1, x1+= Δi/10)
    for (x2 = ...
        y = f(x1, x2);
        if y > y rec then yrec = y;
```

<u>We have</u>

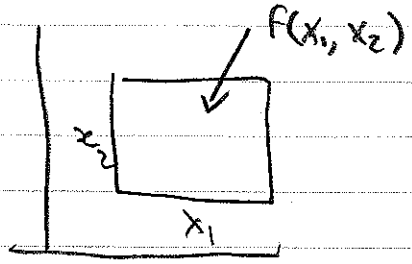
$y = f(x_1, x_2)$

$x_1 \in [x_1 - \Delta_1, x_1 + \Delta_1]$

$x_2 \in [x_2 - \Delta_2, x_2 + \Delta_2]$



- we want to find the large possible value.
- " " " smallest "



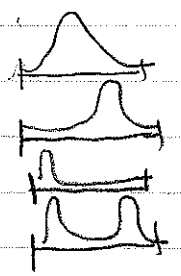
Problem

① Same as statistical

When $n$ is large, we have too many calls to $f$

② linearization is an approximate tool. → in general the problem is NP-Hard

We can have



ideally we need ≈ Monte Carlo method.

problem: $\sum_{i=1}^{n} \frac{\partial f}{\partial x_i} \Delta x_i$

$$\sigma^2 = \sqrt{\sum \left(\frac{\partial f}{\partial x_i}\right)^2 \cdot \sigma_i^2}$$

Example $y = f(x_1, \dots, x_n) = x_1 + \dots + x_n$

$$\tilde{y} = \tilde{x}_1 + \dots + \tilde{x}_n$$

$$\Delta y = \tilde{y} - y = (\tilde{x}_1 + \dots + \tilde{x}_n) - (x_1 + \dots + x_n)$$

(we open ( ) ) $= \tilde{x}_1 + \dots + \tilde{x}_n - x_1 - \dots - x_n)$

$$(\tilde{x}_1 - x_1) + (\tilde{x}_2 - x_2) + \dots + (\tilde{x}_n - x_n)$$

$$= \Delta x_1 + \dots + \Delta x_n$$

$$-\Delta_i \le \Delta x_i \le \Delta_i$$

Correct answer

$$\Delta = n \cdot \Delta_1$$

$$\sqrt{\sum \sigma_i^2} \le \sqrt{n \Delta_1^2} = \Delta_1 \sqrt{n}$$

$$\sigma_i^2 \le \Delta_1^2$$

$\boxed{\text{We are underestimating the error!!!}}$

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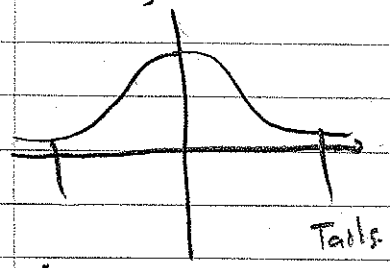
Cauchy distribution,




$$\rho(x) = \frac{1}{\pi} \cdot \frac{1}{1+\left(\frac{x}{\Delta}\right)^2}$$

$$\rho_{Gauss}(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

$\Delta$ never goes to <u>zero</u>

$$V = \sum p_i \lambda_i^2$$

$$\int_{-\infty}^{\infty} \rho(x)\, x^2\, dx$$

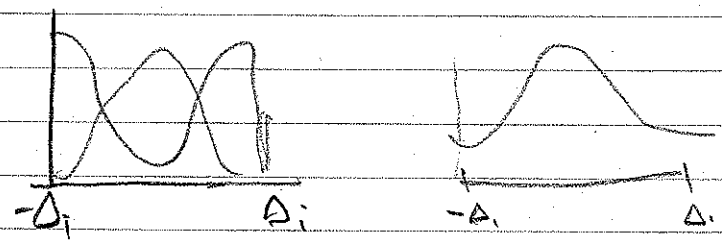
$$\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x^2}{1+\lambda^2}\, dx$$

<u>Gaussian distribution:</u>

$$\Delta y = \sum_{i=1}^{n} c_i \Delta x_i$$

$\Delta x_i$ are Gaussian s.t. dev $\sigma_i$
then $\Delta y$ is also Gaussian w/ st. dev $\sigma = \sqrt{\sum_{i=1}^{n} c_i^2 \sigma_i^2}$

<u>Cauchy distribution</u> $\Delta y = \sum c_i \Delta x_i$
$\Delta x_i$ are Cauchy distr w/ param $\Delta_i$
then $\Delta y$ is also Cauchy distr with parameter $\Delta = \sum_{i=1}^{n} |c_i| \Delta_i$



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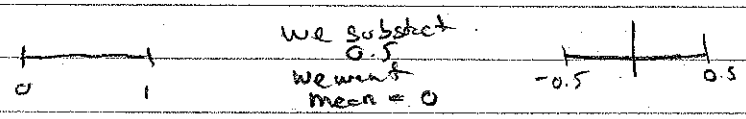
2 Questions.

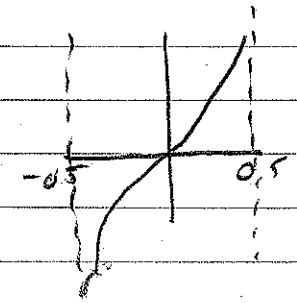
- How to simulate Cauchy distr.
- based on sample how to estimate $\Delta$

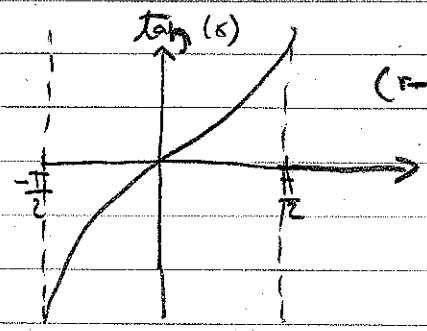
How to simulate cauchy dist.




We want to strech it!!

We need a fcn that looks like this.




$(r-0.5)\cdot\pi$

to obtain a Cauchy random number: $c$

$$c = \tan((\text{rand}()-0.5)\cdot\pi)$$

$\rho(x)\,\Delta x$ the prob is proportional to the