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3 PM BUSN 319
Friday Oct 24^{th}
Inverse Problem and
Newton's method

Typical mistake in the HW.

$$\Delta = \sum_{i=1}^{n} \left| f(\tilde{x}_1, \ldots, \tilde{x}_{i-1} + \Delta_i, \tilde{x}_{i+1}, \ldots, \tilde{x}_n) - \tilde{y} \right|$$

⚠ Forgot the absolute value!!

e.g.,

$f(x_1, x_1) = x_1 - x_2$

$\tilde{x}_1 = 1.0 \qquad \tilde{x}_2 = 1.0$

$\Delta_1 = 0.1 \qquad \Delta_2 = 0.1$

$x_1 \in [0.9, 1.1] \qquad \tilde{y} = 0$

$x_2 \in [-0.2, 0.2]$

$|f(\tilde{x}_1 + \Delta_1, \tilde{x}_2) - 0.0| + |f(\tilde{x}_1, \tilde{x}_2 + \Delta_2)| = \Delta$

$1.1 - 1.0 = \underline{0.1}$

Random error:

- Data Fusion $\tilde{x}_1, \ldots, \tilde{x}_n \rightarrow \tilde{x} = \dfrac{\sigma^{-2}\tilde{x}_1 + \ldots +}{\sigma^{-2} + \ldots +}$
- Data Processing: Several algorithms

Interval error: (only know upper bound, we don't know meas. error)

$\tilde{x}_1 = 1.0$, $\Delta_1 = 0.1 \rightarrow x \in [0.9, 1.1]$

$\tilde{x}_2 = 0.9$, $\Delta_2 = 0.2 \rightarrow x \in [0.7, 1.1]$

$\tilde{x}_3 = 1.07$, $\Delta_3 = 0.05 \rightarrow x \in [1.02, 1.12]$

<u>We take the intersection</u>

$$x \in [1.02, 1.1]$$

$$[\tilde{x} - \Delta, \tilde{x} + \Delta]$$

$\bar{X} = \frac{\underline{x} + \bar{x}}{2}$ midpoint

$\tilde{x} = 1.06$

$\underline{\Delta} = 0.04$

$\Delta = \frac{\bar{X} - \underline{X}}{2}$ half width

<u>Trust</u>

$\tilde{x}_1 = 1.0$ $\Delta_1 = 0.1$; $x \in [0.9, 1.1]$

$\tilde{x}_2 = 0.9$ $\Delta_2 = 0.2$; $x \in [0.7, 1.1]$

<u>In reality</u>:

- errors happen, there is always a prob. that something goes wrong
- We are not 100% sure.

⚠ This is extremely <u>important</u> in the Interval Case.

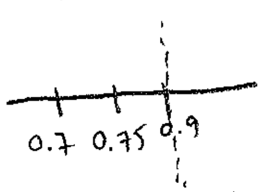
$\tilde{X}_3 = 0.8$ $\Delta_3 = 0.05$ $x[0.75, 0.85]$

$\boxed{\text{Outlier}}$ If we take the intersection $\rightarrow \phi$

💡 We may know the reliability of the measurment.

Case: 2 meas. out of 3 are correct.

$$\begin{cases} 0.9 & 1.1 \\ 0.7 & 1.1 \\ 0.75 & 0.85 \end{cases}$$



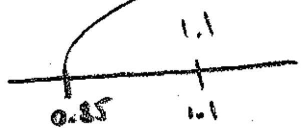
If all were correct, we choose the upper bound: <u>0.9</u>

$x \geq 0.75$

10/23/08

$$\begin{cases} 0.9 \text{ is correct lower bound} \Rightarrow x \geq 0.9 \\ 0.9 \text{ is a fluke, not a correct lower bound} \Rightarrow x \geq 0.75 \end{cases} \Rightarrow x \geq 0.75$$

Same for the:



$$\left.\begin{array}{l} x \leq 0.85 \\ \text{or} \\ x \leq 1.1 \end{array}\right\} \Rightarrow x \leq 1.1$$

$[0.75, 1.1]$

The algorithm is the following:

$\tilde{x}_i$, Δ_i (P) probability of correctness (reliability or trust)

$1 \leq i \leq n$

$\underline{x}_1 = \tilde{x}_1 - \Delta_1, \ldots, \underline{x}_n = \tilde{x}_n - \Delta_n$

$\underline{x}_{(1)} \leq \underline{x}_{(2)} \leq \ldots \leq \underline{x}_{(n)}$

$\underline{x}_1 = 0.7$

$\underline{x}_2 = 0.75$

$\underline{x}_3 = 0.9$

$\underline{x}_{(1)} = 0.7 \leq \underline{x}_{(2)} = 0.75 \leq \underline{x}_{(3)} = 0.9$

We take $\underline{x}_{(n \cdot p)}$

$\overline{x}_i = \tilde{x}_i + \Delta_i$

$\overline{x}_{(1)} \leq \overline{x}_{(2)} \leq \ldots \leq \overline{x}_{(n)}$

$\overline{x}_{(n \cdot (1-p))}$

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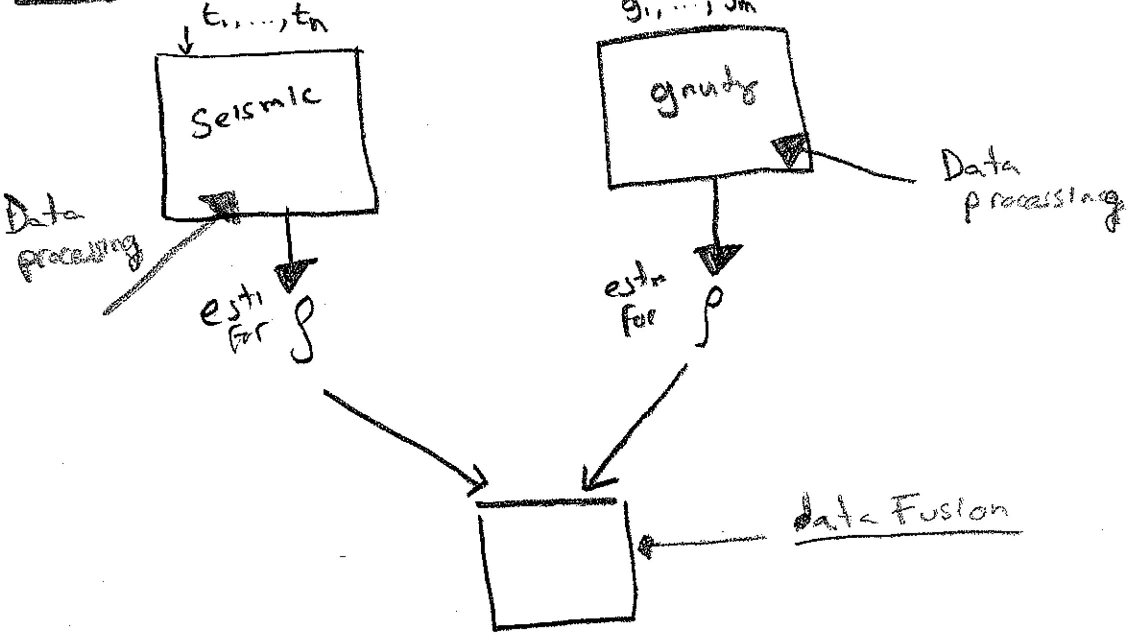
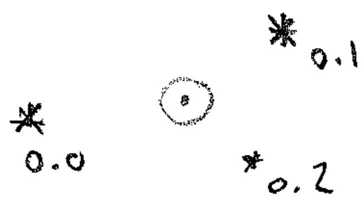
In Cyber-Infrastructure



Trust

- data fusion
- data processing

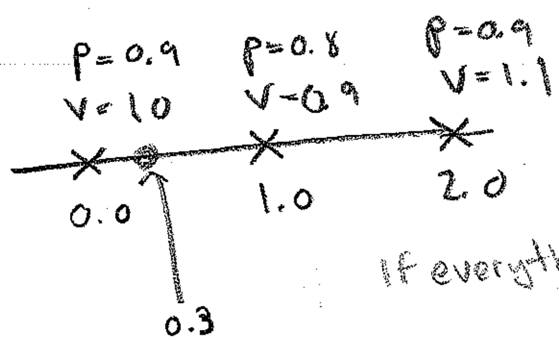
Toy example.



motivations:

Simplest way: K-nearest neighbor.

1-NN



If everything is perfect $\Rightarrow$

	prob
-1.0	0.9
-0.9	0.08
-1.1	0.018
?	