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1) What problem are solving?

Given. $\tilde{x}_1, \ldots, \tilde{x}_n$

$\Delta_1, \ldots, \Delta_n$

$F(x_1, \ldots, x_n)$

We need Δ

So far

- Uncertainty
  - Data processing
    - random error w/ know char — 3 → # of algorithms: ① analytical ② Numerical ③ Monte Carlo
    - upper bound — 3 → ① analytical ② Numerical ③ Monte Carlo (Cauchy)
  - Data Fusion
    - random error w/ known characteristics — 1
    - upper bounds — 1
- Trust — 1 ½
  - 1 Monte Carlo
  - Analytical prob.

2) Why Monte-Carlo

$$\sigma = \sqrt{\sum^{n} \left(f(\tilde{x}_1, \ldots, \tilde{x}_{i-1}, \tilde{x}_i + \sigma, \tilde{x}_{i+1}, \ldots, \tilde{x}_n)^2 - \tilde{y} \right)^2}$$

⚠ Computin f takes too much time.

Other methods: $n+1$ calls to f. where n is # of inputs

Monte Carlo: If we run it N times we get accuracy

$$\sim \frac{1}{\sqrt{N}} \qquad \frac{1}{\sqrt{N}} = 10\% = 0.1 = \frac{1}{10}$$

$$\sqrt{10} = 10$$

$$N \approx 100$$

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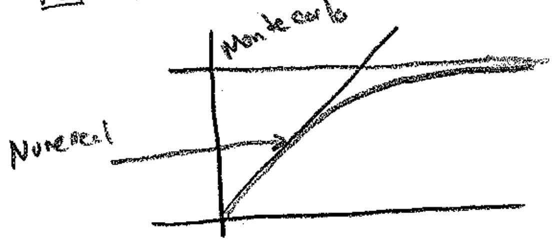
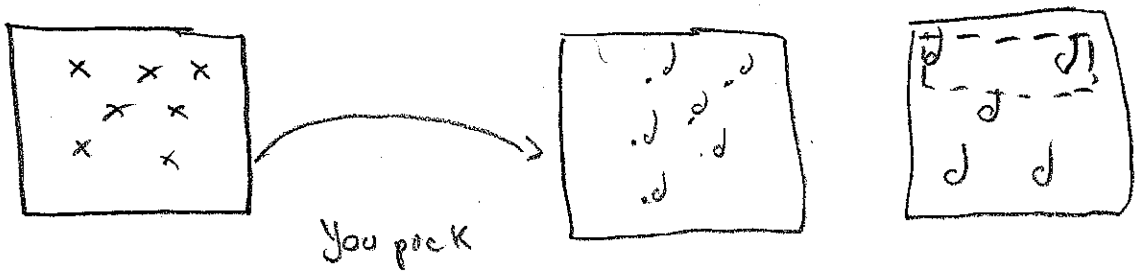
⚠ There are other methods.



⚠ Send e-mail to Dr Kreinovich in case you want that Quantum Comp. be offered !!

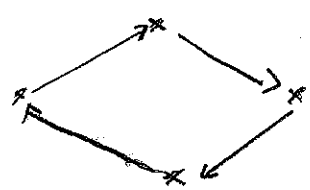
(3) What are we doing with Trust? (HW by Tue)

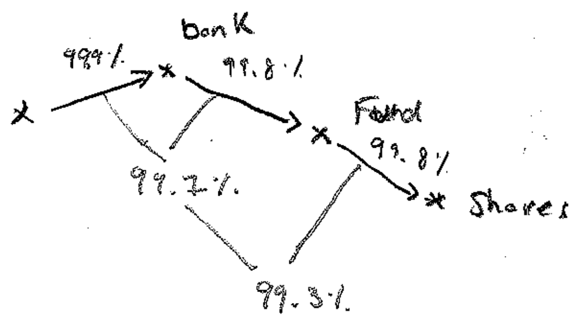


Trust (reliability)

Basic algorithm: Monte Carlo Simulation

e.g.,





Problem

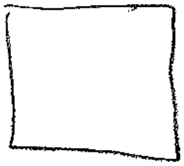
$$\frac{1}{\sqrt{N}} \approx 0.1\% = \frac{1}{1000}$$

$$\boxed{N \approx 1{,}000{,}000}$$

1.69 ± 1 cm

5' 7" ± 1/2" σ, Δ

The whole purpose: 100 iterations is enough!



- Chip has a lot of resistors.
- We want to know the reliability of this chip.

10^{-6}

0.0001%.

<u>Problem:</u> when the probabilities $p_1, \ldots, p_n$ are close to 1, we need too many iterations to compute p by: Monte Carlo simulations.

~~Question:~~ <u>What can we do?</u>

<u>Idea of the solution.</u> "re-scale" p_1 and then "re-scale" the results back.

<u>Details:</u> to follow

Suppose we have a simple chain of events

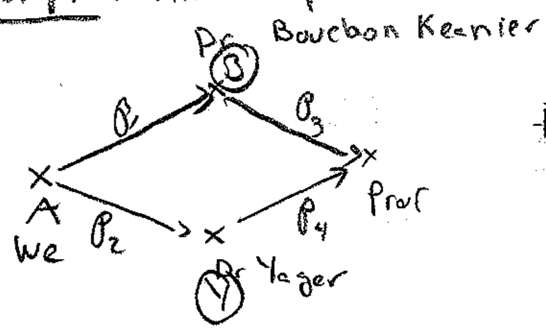
$$\underset{A}{\times} \xrightarrow{p_1} \underset{B}{\times} \xrightarrow{p_2} \underset{C}{\times}$$

C to be trusted..

trust (A in B) is correct ← p_1

& trust (B in C) is correct ← p_2

<u>Another example</u>: hire a professor.



$$\overbrace{\text{trust (We in B) \& trust (B in M)}}^{p_1 \cdot p_3}$$

$$P(\neg A) = 1 - P(A)$$

$$A \vee B \equiv \neg(\neg A \,\&\, \neg B)$$

$$P(\neg A) = 1-a \qquad P(\neg B) = 1-b$$

$$P(A \vee B) = 1 - (1-a)(1-b) =$$

$$= \cancel{1} - \cancel{1} + a + b - a \cdot b$$

$$a + b - a \cdot b$$

$$P(A \vee B) = P(A) + P(B) - P(A \wedge B)$$

$$P_1 \cdot P_3 + P_2 \cdot P_4 - (1 - P_1 \cdot P_3)(1 - P_2 P_4)$$

Y	$S_1 \; S_2 \; S_3 \; S_4$	$P_1 \cdot P_2 \cdot P_3 \cdot P_4$
Y	$S_1 \; S_2 \; S_3 \; \overline{S_4}$	$P_1 \cdot P_2 \cdot P_3 \cdot (1 - P_4)$
Y	$S_1 \; S_2 \; \overline{S_3} \; S_4$	$P_1 \cdot P_2 \cdot (1 - P_3) \cdot P_4$
N	$S_1 \; S_2 \; \overline{S_3} \; \overline{S_4}$	$P_1 \cdot P_2 \cdot (1 - P_3)(1 - P_4)$
Y	$S_1 \; \overline{S_2} \; S_3 \; S_4$	$P_1 \cdot (1 - P_2) P_3 \cdot P_4$
Y	$S_1 \; \overline{S_2} \; S_3 \; \overline{S_4}$	$P_1 \cdot (1 - P_2) P_3 \cdot (1 - P_4)$
N	$S_1 \; \overline{S_2} \; \overline{S_3} \; S_4$	$P_1 \cdot (1 - P_2) \cdot (1 - P_3) P_4$
N	$S_1 \; \overline{S_2} \; \overline{S_3} \; \overline{S_4}$	$P_1 \cdot (1 - P_2)(1 - P_3) \cdot (1 - P_4)$

$$P_1 \cdot P_2 \cdot P_3 \cdot P_4 + P_1 \cdot P_2 \cdot (1 - P_3) P_4 + \ldots$$

In General = polinomial of P_1

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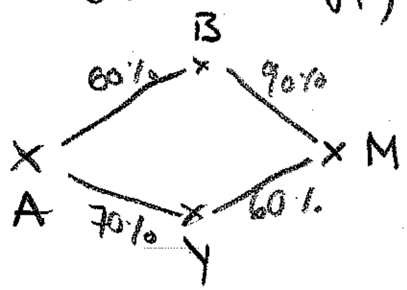
$P = f(P_1, \ldots, P_n)$ → polynomial

<u>Problem</u> $P_i = 1 - \Delta P_i$, $\Delta P_i \ll 1$

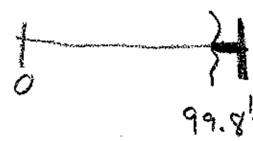


$\frac{1}{\sqrt{N}}$

99%

100 ± 10 %

99.8%



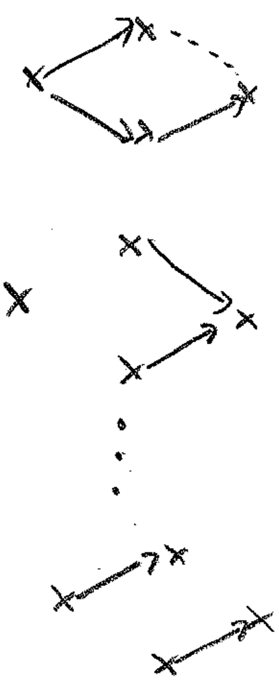
Everytime you have a link
you decide if you add it or not

e.g.,

1)

2)

N)



MonteCarlo

Same thing
as the maps!

⚠ This is the case they are
independent.

Problem $\beta_1 = 1 - \Delta\beta_i\,,\quad \Delta\beta_1 \ll 1$

Idea $\left\{\begin{array}{l} 1-\beta = 1 - f(\beta_1, \ldots, \beta_n) = \\ \qquad 1 - f(1-\varepsilon\Delta\beta_1, \ldots, 1-\varepsilon\cdot\Delta\beta_n) = \text{Polynomial} \end{array}\right\}$ Confusing !!

$K \gg 1$

$\Delta\beta_i = \delta\cdot\Delta\beta_i' \qquad \Delta\beta_i' = \boxed{K\cdot\Delta\beta_i}$

$\delta \ll 1$

$$\boxed{1-\beta = f(1-\delta\Delta\beta_1', \ldots, 1-\delta\Delta\beta_n') = \text{Polynomial of } \delta}$$

$$\beta = \beta_1\cdot\beta_2 = f(\beta_1, \beta_2) =$$

$$1-\beta = 1-(1-\delta\Delta\beta_1)(1-\delta\Delta\beta_2) =$$

$$= f(\Delta\beta_1' + \Delta\beta_2') - \delta^2\cdot\Delta\beta_1'\cdot\Delta\beta_2'$$

$$\underset{=0}{a_0} + a_1\delta + a_2\delta^2 + \ldots +$$