Interval Computations

* Monotonic Case
* Linearized Case
* Numerical diff technique
* Cauchy-based method (w/ derivation)

Reliability and trust

Statistical
* General idea Monte-Carlo est
* analytical expression Z-1 NN.
* re-Scaling Monte-Carlo

Possible dependence
* Analytical f-las for simple case.
  PA — PB → P(A & B)
* Shortest-path alg for trust (w/out derivation)

Intervals as a way to provide privacy

Intervals & privacy

$a_1 \in [10, 20]$ Mid point $\tilde{x} = 15$ half width $\Delta_1 = 5$

$a_2 \in [20, 30]$

Mean

$$M = \frac{a_1 + a_2}{2}$$

Variance

$$V = \frac{a_1^2}{2} + \frac{a_2^2}{2} - \left(\frac{a_1 + a_2}{2}\right)^2$$

$$f(x_1, x_2) = \frac{x_1 + x_2}{2}$$

$$\Delta = \sum_{i=1}^{n} \left|\frac{\partial f}{\partial x_i}\right| \Delta_i = \frac{1}{2} \cdot \Delta + \frac{1}{2} \Delta_2 = 5$$

$$M = [15, 25]$$

For the Variance

$$\frac{15^2}{2} + \frac{25^2}{2} - 20^2 = \frac{225}{2} + \frac{625}{2} - 400 = 425 - 400 = \underline{25}$$

$$f(x_1, x_2) = \frac{x_1^2}{2} + \frac{x_2^2}{2} - \left(\frac{x_1 + x_2}{2}\right)^2$$

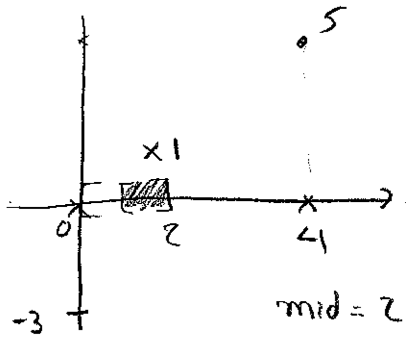
$$\frac{\partial f}{\partial x_1} = \frac{\cancel{2} x_1}{\cancel{2}} - 2\left(\frac{x_1 + x_2}{2}\right)\left(\frac{1}{2}\right) = \frac{x_1 - x_2}{2} = \underline{-5}$$

$$\frac{\partial f}{\partial x_2} = \frac{x_2 - x_1}{2} = \underline{5}, \quad \Delta = |-5| \cdot 5 + |5| \cdot 5 = \boxed{50}$$

Bisection

$f(x) = 2x - 3 = 0$

$x \in [0, 4]$

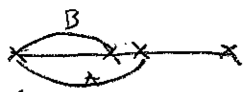


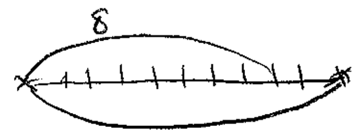
lower = 0	lower = 0	lower = 1
upper = 4	upper = 2	upper = 2
	mid = 1	mid = 3/2
	f(mid) < 0	

Probabilities

: We have two 2 events, describe the range.

A: $PA = 0.8$ B: $PB = 0.7$





$[\max(PA + PB - 1, 0), \min(PA, PB)]$

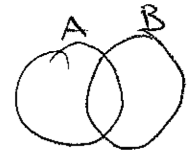
0.8 . 0.7

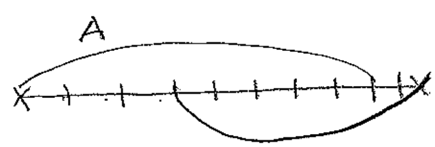
$[0.5, 0.7]$

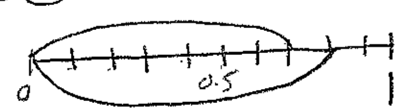
$P(A \vee B) = P(\neg(\underset{0.7}{\neg A} \,\&\, \underset{0.8}{\neg B}))$

$[\max(PA, PB), \min(PA + PB, 1)]$

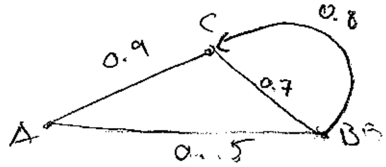
$[0.8, 1]$

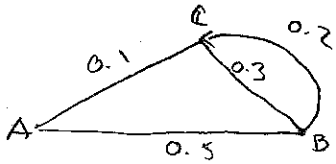






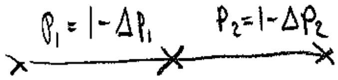
Shortest Path





$$1 - 0.4 = 0.6$$

Re-Scaling (Monte Carlo)

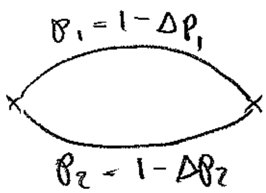
- 

$$p = p_1 \cdot p_2 = (1 - \Delta p_1) \cdot (1 - \Delta p_2) =$$

$$1 - \Delta p_1 - \Delta p_2 + \Delta p_1 \cdot \Delta p_2$$

$$\Delta p = 1 - p = \Delta p_1 + \Delta p_2 - \cancel{\Delta p_1 \Delta p_2}$$

$$\Delta p' = \lambda \Delta p_1 + \lambda \Delta p_2 = \lambda (\Delta p_1 + \Delta p_2)$$

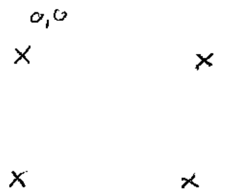
- 

$$p = 1 - \Delta p_1 + 1 - \Delta p_2 - (1 - \Delta p_1)(1 - \Delta p_2) = \Delta p_1 \cdot \Delta p_2$$

$$\Delta p' = (\lambda \Delta p_1)(\lambda \Delta p_2) =$$

$$\lambda^2 (\Delta p_1 \cdot \Delta p_2)$$

Analytical Expression



$x_1 = (0.5, 0.6)$, $V_1 = 1.0$ $P_1 = 0.9$

$x_2 = (0, 1.3)$, $V_2 = 1.3$ $P_2 = 0.8$

$x_3 = (1.3, 1)$, $V_3 = 1.6$ $P_3 = 0.95$

$= 0.5^2 + 0.6^2 = 0.25 + 0.36 = 0.61$

$= 0^2 + 1.3^2 = 1.69$

$= 1.3^2 + 1^2 = 2.69$

0.9	1.0
0.1·0.8=0.08	1.3
0.1·0.2·0.95 0.02	1.6

Mean:

$0.9 \cdot 1.0 + 0.08 \cdot 1.3 + 0.02 \cdot 1.6 =$

$0.9 + 0.10 + 0.03 = \underline{1.03}$

Variance (0,0)

0.9	−0.03	0.001 =	0.001
0.08	0.27	0.08 =	0.006
0.02	0.57	0.36 =	0.007

$\sqrt{0.014} \approx 0.12$

$$V = \frac{x_1^2}{2} + \frac{x_2^2}{2} - \left(\frac{x_1 + x_2}{2}\right)^2$$

$$\frac{\partial f}{\partial x_1} = \frac{x_1 - x_2}{2}$$

$$\frac{\partial f}{\partial x_2} = \frac{x_2 - x_1}{2}$$

$\tilde{x}_1 = 1.0 \quad \Delta_1 = 0.1$

$\tilde{x}_2 = 2.0 \quad \Delta_2 = 0.1$

$$\Delta = \left|\frac{\partial f}{\partial x_1}\right| \cdot \Delta_1 + \left|\frac{\partial f}{\partial x_2}\right| \Delta_2 = 0.5 \cdot 0.1 + 0.5 \cdot 0.1 = \underline{\underline{0.1}}$$

Nominal Value

$$\tilde{y} = \frac{1}{2} + \frac{4}{2} - 1.5^2 =$$

$$0.5 + 2 - 2.25 = 0.25$$

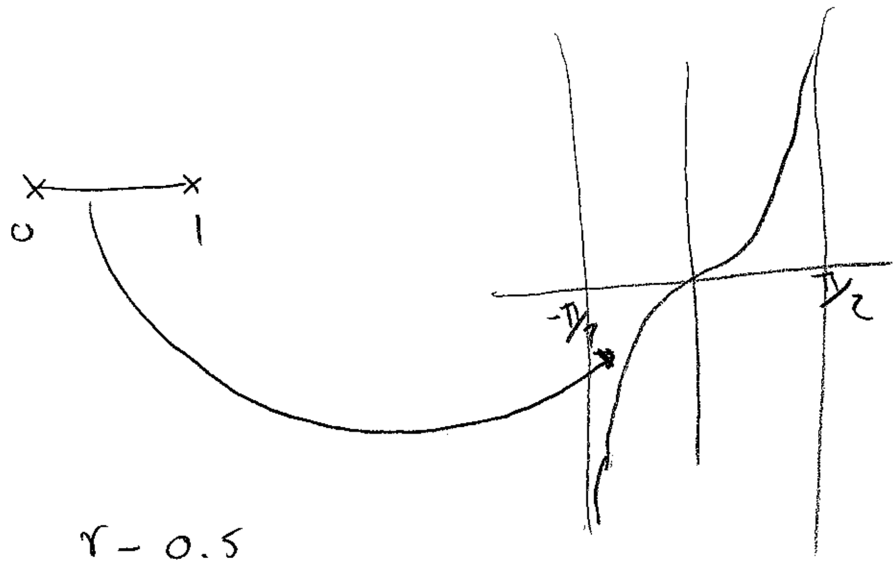
Numerical differentiation

$$\Delta = \Big|\overbrace{f(\underset{1.1}{\tilde{x}_1 + \Delta_1}, \underset{2.0}{\tilde{x}_2})}^{0.2025} - \overset{0.25}{\tilde{y}}\Big| + \Big|\overbrace{f(\tilde{x}_1, \tilde{x}_2 + \Delta_2)}^{0.3025} - \overset{0.25}{\tilde{y}}\Big|$$

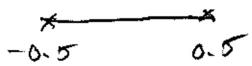
$$\Delta = \frac{1.1^2}{2} + 2 - 1.55^2 = \frac{1.21}{2} + 2 = 2.4025$$

$$0.605 + 2 - 2.4025 = 0.2025$$

- Why we need Monte Carlo? to decrease the computations
- Why we take tangent? [0, 1] we want transformation to have big #'s.



$r - 0.5$



$\tan(\pi(r-0.5))$

△ Comparison with statistical methods vs Correlations.

e.g.,

$f = x_1 + x_2$

Case 1: independent σ_1, σ_2, $\sigma = \sqrt{\sigma_1^2 + \sigma_2^2}$

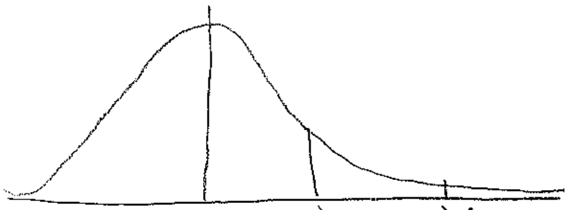
Case 2: Δ_1 Δ_2 $\Delta_1 + \Delta_2$

$y = x_1 + \dots + x_n$ $\sigma = \sqrt{n} \cdot \sigma_i$

$\Delta = n \cdot \Delta_1$

- M.C. ⇒ statistical Case ☑
- N.Diff ⇒ Interval you dont need '$\sqrt{\ }$'

11/18/08

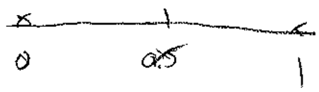


$$\frac{1}{\sqrt{1000}} \approx \frac{1}{30} \approx 3\%$$

$\Delta \leq 3\sigma$

6σ

1 0.5



$V = 0.5$

Std Dev $= \sqrt{N}$

$$E\left(\frac{\Delta x_1 + \dots + \Delta x_n}{n}\right)^2 = \frac{n \cdot E\Delta x_i^2}{n^2} = \frac{V_n}{n}$$

- What problem we are solving: Find the possible range of the indirect measurements.