CS 5354/CS 4365 Advanced Computational Methods in Economics and Finance
Fall 2018, Test 1

Name: __ ________

100/100

General comments:

- you are allowed up to 5 pages of handwritten notes;
- if you need extra pages, place your name on each extra page;
- the main goal of most questions is to show that you know the corresponding algorithms; so, if you are running of time, just follow the few first steps of the corresponding algorithm;

Good luck!

10/10 1. What is utility? Give a precise definition.

10/10 2. How do you select the very bad alternative A_- and the very good alternative A_+ that are needed to define utility?

10/10 3. Once you have selected the very bad and the very good alternatives, how do we find the utility of a given alternative A? Explain, in detail.

Q1

Ans: Utility u of any event 'A' is defined as the probability for which A has same value as the lottery in which one gets a very good result A+ with probability u and a very bad result A⁻ with a probability 1-u.

Q2.

Ans: The very bad Alternative A- should be worse than any possible outcome that we can have and the very good Alternative A+ should be better than any possible outcome we can have.

Q3.

Ans: Once we have selected the very bad and the very good alternatives, we do the following:

Utility calculation is essentially done by bisection. initially utility of any event A $u(A) \in [0,1]$. i.e in interval $[\underline{u}, \overline{u}]$ where $\underline{u} = 0$ & $\overline{u} = 1$.
Now we find the mid point $m = \frac{\underline{u}+\overline{u}}{2}$ and ask the user to compare the alternative A with lottery $L(m)$.

if $A > L(m)$ we say utility $u(A) \in [m, \overline{u}]$
if $A < L(m)$ we say utility $u(A) \in [\underline{u}, m]$

Thus in either case we get an interval of half the width we started with.
Now we continue this process until the resulting interval is sufficiently small. This is how we get the value of utility.

eg. if we start with initial interval $[0,1]$

Now we initially ask the question 1. the estimate of utility $\tilde{u} = \frac{0+1}{2} = 0.5$
Now depending on user's choice the utility either is in interval $[0, 0.5]$ or $[0.5, 1]$

Now the new estimate of utility is either $\frac{0+0.5}{2} = 0.25$ or $\frac{0.5+1}{2} = 0.75$ depending on users answer to question 1.
Again we ask question 2.
So the utility would be somewhere in interval $[0, 0.25]$ or $[0.25, 0.5]$ or $[0.5, 0.75]$ or $[0.75, 1]$ depending on user's answer to the two question's asked in this way by narrowing the interval we find utility.

10/10 4. With what accuracy can we determine the utility if we only ask 3 questions? Explain your answer.

10/10 5. How many questions do you need to ask a person to determine his/her utility with accuracy 10%? Explain your answer.

Q 4.

Ans: initially utility $u(A) \in [0,1]$ i.e $[\underline{u}, \overline{u}]$ where $\underline{u} = 0$ & $\overline{u} = 1$

Now the estimate of utility without asking any questions is

$$\tilde{u} = \frac{0+1}{2} = 0.5$$

Thus the Accuracy $\Delta_0 = |\overline{u} - \tilde{u}|$ or $|\underline{u} - \tilde{u}| = |1-0.5|$ or $|0-0.5|$
$= 0.5 = 1/2^1$

After asking 1 question utility is either in interval $[0.5, 1]$ or $[0, 0.5]$

Thus the New estimate of utility is $\tilde{u} = \frac{0+0.5}{2} = 0.25$

Thus Accuracy Δ_1 after 1 question $= |0.5 - 0.25| = 0.25 = 1/4 = 1/2^2$

Now Accuracy Δ_2 after 2 questions $= 1/2^3$

Simillarly

Accuracy Δ_3 after 3 questions $= 1/2^4 = 1/16 = \underline{\underline{6.25\%}}$

* Note: In general we have After k questions the accuracy $\Delta_k = \frac{1}{2^{k+1}}$

Q5

Ans: from the * note in question 4 we see that After k questions

Accuracy $\Delta u = \frac{1}{2^{k+1}}$

Thus for Accuracy of 10% we must have

$$\frac{10}{100} \geq \frac{1}{2^{k+1}}$$

$$\frac{1}{10} \geq \frac{1}{2^{k+1}}$$

The minimum possible value of k that satisfies the above condition is 3. Thus for accuracy of 10% we have to ask at-least 3 questions.

10/10

6. Suppose that an alternative A has utility u(A) = 0.7 with respect to the original pair (A_-, A_+), and that with respect to a new pair (A'_-, A'_+), we have $u'(A_-) = 0.1$ and $u'(A_+) = 0.9$. What is the utility u'(A) of the alternative A with respect to the new pair? Explain your answer.

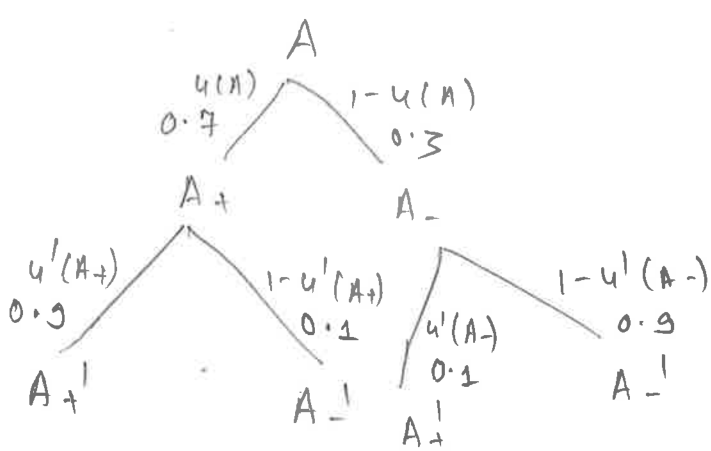
Ans:

Here $A'_- < A_- < A_+ < A'_+$

A is equivalent to lottery in which we get A_+ with probability 0.7 and A_- with remaining probability 0.3

A_+ is equivalent to lottery in which we get A'_+ with probability 0.9 and A'_- with remaining probability 0.1

A_- is equivalent to lottery in which we get A'_+ with probability 0.1 and A'_- with remaining probability 0.9



$u'(A) = 0.7 * 0.9 + 0.3 * 0.1$

$= 0.63 + 0.03$

$= \underline{\underline{0.66}}$

7-9. Suppose that:

- for alternative A, the utility is from 0.5 to 0.7,
- for alternative B, it is from 0.4 and 0.8, and
- for alternative C, it is from 0.1 to 0.9.

Which of these three alternatives will be selected:

10/10 • by a perfect optimist, for whom the Hurwicz optimism-pessimism parameter is $\alpha_H = 1$?
10/10 • by a perfect pessimist, for whom the Hurwicz optimism-pessimism parameter is $\alpha_H = 0$?
10/10 • by a realist, for whom the Hurwicz optimism-pessimism parameter is $\alpha_H = 0.6$?

Q 7:
Ans:

Alternative	utility interval	$\alpha_H = 1$	$\alpha_H = 0$	$\alpha_H = 0.6$
A	[0.5, 0.7]	0.7	0.5	0.6*0.7 + 0.4×0.5 = 0.62
B	[0.4, 0.8]	0.8	0.4	0.6*0.8 + 0.4×0.4 = 0.64
C	[0.1, 0.9]	0.9	0.1	0.6 * 0.9 + 0.4 * 0.1 = 0.58

According to Hurwicz optimism-pesimism principle for any optimism-pesimism paramete α_H Alternative A is better than Alternative B if and only if

$$\alpha_H(A_{max}) + (1-\alpha_H)(A_{min}) > \alpha_H(B_{max}) + (1-\alpha_H)(B_{min})$$

Thus a perfect optimist completely ignores all but the best case scenario and the highest utility value in this case is for option C so perfect optimist chooses C

A perfect pesimist ignores all but worst case scenario and the highest utility value in such case is for option A Thus pesimist choses option A.

Realist chooses option B as the utility value is highest for option B as per Hurwicz criteria.

10/10

10. Explain, in detail, how we can deduce the Hurwicz optimism-pessimism formula from the requirement that decision making under interval uncertainty should not change if we re-scale the utility values.

Ans: The main idea behind Hurwicz's solution to decision making under interval uncertainity is we need to assign single utility value u to each utility interval $[\underline{u}, \overline{u}]$. Since utility is defined modulo linear re-scaling $u \longrightarrow k * u + b$ should not change the assignment.

initially $\alpha_H \in [0,1]$

and utility is defined modulo linear

$$k * \alpha_H + b = f(\underline{u}, \overline{u}) \quad -(1)$$

re-scaling we have

$$\underline{u} = k * 0 + b \Rightarrow \underline{u} = b \quad -(ii)$$
$$\overline{u} = k * 1 + b \Rightarrow \overline{u} = k + b$$
$$\Rightarrow k = \overline{u} - b$$
$$\Rightarrow k = \overline{u} - \underline{u} \quad (iii)$$

$$f(\underline{u}, \overline{u}) = f(k * 0 + b, k * 1 + b)$$

from (1) (ii) & (iii) we have

$$f(\underline{u}, \overline{u}) = (\overline{u} - \underline{u}) * \alpha_H + \underline{u}$$

$$f(\underline{u}, \overline{u}) = \overline{u}\alpha_H - \underline{u}\alpha_H + \underline{u}$$

$$\boxed{f(\underline{u}, \overline{u}) = \overline{u} * \alpha_H + (1 - \alpha)\underline{u}}$$

which is the required Hurwicz optimism-pessimism formula.