# CS 5354/CS 4365 Advanced Computational Methods in Economics and Finance
# Fall 2018, Test 1

100/100

Name: __________

General comments:

- you are allowed up to 5 pages of handwritten notes;
- if you need extra pages, place your name on each extra page;
- the main goal of most questions is to show that you know the corresponding algorithms; so, if you are running of time, just follow the few first steps of the corresponding algorithm;

Good luck!

10/10 1. What is utility? Give a precise definition.

10/10 2. How do you select the very bad alternative $A_-$ and the very good alternative $A_+$ that are needed to define utility?

10/10 3. Once you have selected the very bad and the very good alternatives, how do we find the utility of a given alternative A? Explain, in detail.

1. We select two alternatives - a very good one $A_+$ and a very bad one $A_-$. For each value p from interval [0, 1], we can define a lottery L(p), in which we get the $A_+$ with probability p and $A_-$ with probability (1-p). For each alternative A, there exists a probability p for which the corresponding lottery L(p) is equivalent to A. This probability is called the utility of alternative A and is denoted by u(A).

2. $A_-$ should be worse than anything that we will actually encounter and $A_+$ should be better than anything that we will actually encounter

3. We do it by bisection. At first, utility is in the interval $[0, 1]$, i.e., in $[u_-, u_+]$ where $u_- = 0$ $u_+ = 1$. Once we know such an interval containing $u(A)$, we take the midpoint $m$ and ask the user to compare the alternative $A$ with the lottery $L(m)$.

- If $A$ is better than $L(m)$, $u(A)$ belongs to $[m, u_+]$;
- If $L(m)$ is better than $A$, $u(A)$ belongs to $[u_-, m]$

In both cases, we get the interval halved. Then we continue the procedure with the new interval until the resulting interval becomes sufficient small, or reaches the predefined precision.

10/10 4. With what accuracy can we determine the utility if we only ask 3 questions? Explain your answer.

10/10 5. How many questions do you need to ask a person to determine his/her utility with accuracy 10%? Explain your answer.

4. We start with the interval with ~~length~~ width 1. After each question, the width of the interval halves. So after 3 questions, we get an interval of width $\frac{1}{2^3} = \frac{1}{8}$. Then we continue to take the midpoint of this interval. The difference between this point and any value from this interval does not exceed half width $\frac{1}{2} \cdot \frac{1}{8} = \frac{1}{16}$.

So, after 3 questions, we can determin the utility with accuracy $\frac{1}{2^4}$

5. Due to the same logic, after k questions, we can determine the utility with accuracy $\frac{1}{2^{k+1}}$. If we want accuracy 10%, we need to find the smallest k for which $\frac{1}{2^{k+1}} < \frac{10}{100}$, i.e., $2^{k+1} > \frac{100}{10}$

$\because 2^4 = 16 > 10$.

$\therefore k+1 = 4 \quad k = 3$.

Therefore, we need to ask 3 questions to determine the utility with accuracy 10%.

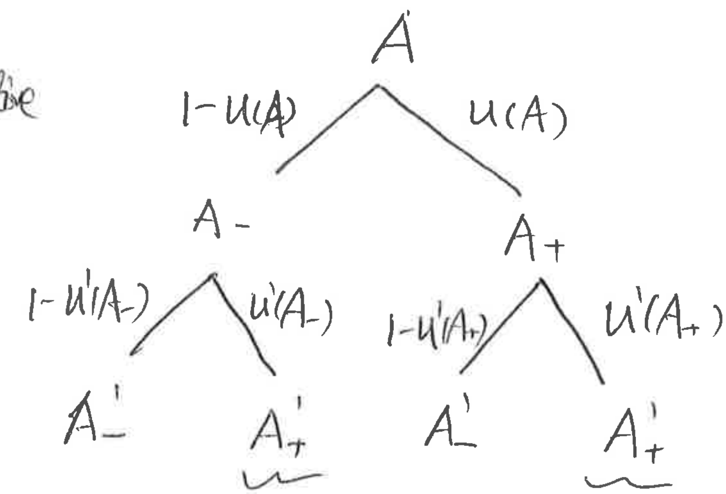
10 | 10

6. Suppose that an alternative A has utility u(A) = 0.7 with respect to the original pair ($A_-$, $A_+$), and that with respect to a new pair ($A'_-$, $A'_+$), we have u'($A_-$) = 0.1 and u'($A_+$) = 0.9. What is the utility u'(A) of the alternative A with respect to the new pair? Explain your answer.

$A'_- < A_- < A_+ < A'_+$

To get the very very good alternative $A'_+$ from A, we have two brances.

The probability is the sum of these two.




$$u'(A) = u(A) \cdot u'(A_+) + (1-u(A)) \cdot u'(A_-)$$
$$= 0.7 \times 0.9 + 0.3 \times 0.1$$
$$= 0.63 + 0.03$$
$$= 0.66$$

7-9. Suppose that:

- for alternative A, the utility is from 0.5 to 0.7,
- for alternative B, it is from 0.4 and 0.8, and
- for alternative C, it is from 0.1 to 0.9.

Which of these three alternatives will be selected:

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- by a perfect optimist, for whom the Hurwicz optimism-pessimism parameter is $\alpha_H = 1$?

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- by a perfect pessimist, for whom the Hurwicz optimism-pessimism parameter is $\alpha_H = 0$?

10/10
- by a realist, for whom the Hurwicz optimism-pessimism parameter is $\alpha_H = 0.6$?

~~$\alpha_H = 1$~~ By Hurwicz optimism-pessimism formula.

If $\alpha_H A_{max} + (1-\alpha_H)A_{min} > \alpha_H B_{max} + (1-\alpha_H) B_{min}$. we select A. otherwise. B is better than A. we select B.

1). $\alpha_H = 1$. $f[0.5, 0.7] = 0.7$. (equivalent utility of A).

$f[0.4, 0.8] = 0.8$ $f[0.1, 0.9] = 0.9$

So alternative C will be selected.

2). $\alpha_H = 0$. $f[0.4, 0.8] = 0.4$ $f[0.5, 0.7] = 0.5$ $f[0.1, 0.9] = 0.1$.

So alternative A will be selected.

3) $\alpha_H = 0.6$. $f[0.5, 0.7] = 0.6 \times 0.7 + 0.4 \times 0.5 = 0.62$.

$f[0.4, 0.8] = 0.6 \times 0.8 + 0.4 \times 0.4 = 0.64$

$f[0.1, 0.9] = 0.6 \times 0.9 + 0.4 \times 0.1 = 0.58$.

So alternative B will be selected.

10/10

10. Explain, in detail, how we can deduce the Hurwicz optimism-pessimism formula from the requirement that decision making under interval uncertainty should not change if we re-scale the utility values.

Suppose $u_{equiv} \in [\underline{u}, \bar{u}]$. and $u' = k * u + b$

Then $k\, u_{equiv} + b = f[k\underline{u} + b, k\bar{u} + b]$.

Particularly. if $\underline{u} = 0$ and $\bar{u} = 1$. i.e. $\alpha_H \overset{equiv}{=} f[0,1]$

we have $k\alpha_H + b = f[b, k+b]$

we want $b = \underline{u}$ $k+b = \bar{u}$ $\Rightarrow b = \underline{u}$ $k = \bar{u} - \underline{u}$

$f[\underline{u}, \bar{u}] = (\bar{u} - \underline{u})\alpha_H + \bar{u} = \bar{u}\alpha_H + (1-\alpha_H)\underline{u}$

Given any $\alpha_H$. we can say alternative A is better than alternative B

if $\alpha_H A_{max} + (1-\alpha_H) A_{min} > \alpha_H B_{max} + (1-\alpha_H) B_{min}$

where $I_{max}$ is the largest utility of I

$I_{min}$ is the smallest utility of I. $I = A, B$.