CS 5354/CS 4365 Advanced Computational Methods in Economics and Finance
Fall 2018, Test 2

Name: ____ ______

General comments:

- you are allowed up to 5 pages of handwritten notes;
- if you need extra pages, place your name on each extra page;
- the main goal of most questions is to show that you know the corresponding algorithms; so, if you are running of time, just follow the few first steps of the corresponding algorithm;

Good luck!

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1. Which of the two actions is better: A or B? For action A, there are two possible outcomes: a_1 with probability 0.4 and utility 50, and a_2 with probability 0.6 and utility −20. For action B, there are also two possible outcomes: b_1 with probability 0.3 and utility 40, and b_2 with probability 0.7 and utility −10.

Let E(A) and E(B) be the eepected utility of action A and B

$E[A] = (.4 \times 50) + (.6 \times -20) = 8$

$E[B] = (.3 \times 40) + (.7 \times -10) = 5$

Since $E(A) > E[B]$, Action A is better

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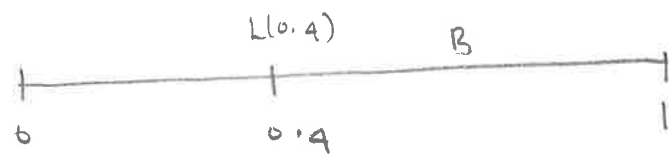
2. To determine the subjective probability that a student S will get A on this test we asked his colleague C whether he prefers \$100 with probability 0.4 or \$100 if S gets an A. The student C selected \$100 if S gets an A. Based on this answer, what can we conclude about C's subjective probability that S will get an A in this test? To be more precise, based on this answer, what is the interval of possible values of the corresponding subjective probability?

Let L = Event that S get A, B = C get \$100 if S get A

Given $L(0.4) = \$100$ and C Selected \$100 if S get A

$\Rightarrow$

$L(0.4) = \$100 < B$



$\therefore \; P_s(L) \in (0.4, 1]$

ie subjected probability is between 0.4 and 1

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3a. Suppose that a decision maker follows McFadden's formula with β = ln(3). If we have three alternatives, with utilities 1, 2, and 3, with what probability will the expert select each of these three alternatives?

3b. What property was used to justify McFadden's formula?

3 (a) Let $u_1 = 1$, $u_2 = 2$ and $u_3 = 3$ be the utilities of alternatives a_1, a_2, a_3

McFadden's with $\beta = \ln(3)$

$$P(a_i) = \frac{3^{u(a_i)}}{\sum_{i=1}^{3} 3^{u(a_i)}}$$

$$P(a_1) = \frac{3^1}{3^1 + 3^2 + 3^3} = \frac{3}{39}$$

$$P(a_2) = \frac{3^2}{3^1 + 3^2 + 3^2} = \frac{9}{39}$$

$$P(a_3) = \frac{3^3}{3^1 + 3^2 + 3^3} = \frac{27}{39}$$

3 (b) Formular assumes that the function $F(u)$ is shift invariant or probabilities do not change if we increase all utilities by a constant.

iè $F(u+b) = f(b)\,F(u)$

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4a. If we have three alternatives, with utilities 1, 2, and 3, with what probability will the expert select each of the three alternatives is this expert uses a power-function modification of McFadden's formula: with the first powers? with the second powers?

4b. What property was used to justify the power-function modification of McFadden's formula?

4(a) Let $u_1 = 1$, $u_2 = 2$, $u_3 = 3$, utilities of 3 alternatives.

First power

$$P(a_i) = \frac{u(a_i)}{\sum_{i=1}^{n} u(a_i)}$$

$$P(a_1) = \frac{1}{1+2+3} = \frac{1}{6}$$

$$P(a_2) = \frac{2}{1+2+3} = \frac{2}{6}$$

$$P(a_3) = \frac{3}{1+2+3} = \frac{3}{6}$$

Second Power : $P(a_i) = \dfrac{u(a_i)^2}{\sum_i u(a_i)^2}$

$$P(a_1) = \frac{1^2}{1^2+2^2+3^2} = \frac{1}{14}$$

$$P(a_2) = \frac{2^2}{1^2+2^2+3^2} = \frac{4}{14}$$

$$P(a_3) = \frac{3^2}{1^2+2^2+3^2} = \frac{9}{14}$$

4 (b) Modified formula assumes the function $F(u)$ is scale invariant i.e. $F(ku) = f(k)\,F(u)$

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5a. Suppose that a group of two people needs to select between two alternatives. For the first alternative, their utilities are 50 and 10; for the second alternative, their utilities are 70 and 8. Which alternative should they select if they use Nash's bargaining solution?

5b. What property was used to justify Nash's bargaining solution?

5(a)
Let A, B be 1st and 2nd alternatives respectively

$\prod u_A = 50 \times 10 = 500$

$\prod u_B = 70 \times 8 = 560$

Since $\prod u_B > \prod u_A$ by Nash's bargaining solution alternative B should be selected.

5(b) Property used is scale invariant

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6. Give an example that Nash's bargaining solution provides a more fair description of the country's economic situation than the Gross Domestic Product (GDP).

	Country I	Country II
	$u_1 = 100$	$u_1 = 10$
	$u_2 = 1$	$u_2 = 10$
	$u_3 = 1$	$u_3 = 10$
$GDP = \sum u_i$:	102	30
$NBG = \prod_i$:	10^2	10^3

country with 3 people is not very realistic

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7a. By using the fact that utility is proportional to the square root of money, explain why most people prefer \$64 cash to a lottery in which they get \$100 with probability 0.64.

7b. Give an example explaining why the simplified assumption -- that utility is proportional to money amount -- does not lead to a reasonable behavior.

7c. What properties were used to justify the power-law dependence of utility on money?

7(a) Given: $u(m) = \sqrt{m}$

Let: $A = u(64) = \sqrt{64} = 8$

Let B = Expected utility of $L(0.64) = 100$

$B = 0.64 \times \sqrt{100} + 0.36\sqrt{0} = 6.4$

Since $A = 8 > B = 6.4$, that is why \$64 is prefered.

x (7b) Suppose we have two altenative, \$50 in cash and and a Lottery which gives \$100 with probability 0.6

Assume utility is proportional to money ie

$u(m) = m$

$u(50) = 50$ and $u(L(0.5)) = 0.6 \times 100 + 0.4(0)$
$= 60$

ie people will prefer $L(0.6) = \$100$ to \$50 Cash, which is not true in reality, since most people will prefere \$50 Cash to $L(0.6) = \$100$, Avoid the risk

(7c) Assumed that utility of money is scale invariant

ie $u(km) = f(k)\,u(m)$

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8. Provide the detailed proofs of the three main mathematical results that we had:

- that every shift-invariant function F(x), i.e., a function for which F(x + c) = f(c) * F(x) for some f(c), has the form F(x) = const * exp(β * x);
- that every scale-invariant function F(x), i.e., a function for which F(k * x) = f(k) * F(x) for some f(k), has the form F(x) = const * x^α;
- that every additive function, i.e., a function for which F(x + y) = F(x) + F(y), has the form F(x) = const * x.

(i) Given $F(x+c) = f(c)\, F(x)$

differentiating w.r.t c

$$\frac{d}{dx} F(x+c) \cdot \frac{d(x+c)}{dc} = \frac{d f(c)}{dc} \cdot F(x)$$

$$\frac{d}{dx} F(x+c) = f'(c)\, F(x) \quad , \quad \text{Let } c = 0$$

$$\frac{d}{dx} F(x) = f'(0)\, F(x) \quad , \quad \text{Let } \beta = f'(0)$$

$$\int \frac{\frac{d}{dx} F(x)}{F(x)} = \int \beta \, dx$$

$$\ln F(x) = \beta x + c$$

$$\therefore F(x) = e^{\beta x} \cdot e^{c} \quad , \quad \text{Let } e^{c} \text{ Constant}$$

$$F(x) = \text{Const } e^{\beta x}$$

(ii) Given $F(kx) = f(k)\, F(x)$, differentiating w.r.t. k

$$\frac{d}{dx} F(kx) \frac{d(kx)}{dk} = \frac{d}{dk} f(k)\, F(x)$$

$$\frac{d}{dx} F(kx)\, x = f'(k)\, F(x) \quad , \quad \text{Let } k = 1 \text{ and } f'(1) = \alpha$$

$$\frac{d}{dx} F(x) \cdot x = \alpha\, F(x)$$

P.T.O

$$\int \frac{dF(x)}{F(x)} = \int \frac{\alpha \, dx}{x}$$

$$\ln F(x) = \alpha \ln x + c$$

$$F(x) = e^{\ln x^{\alpha}} \cdot e^{c}, \quad \text{Let Const} = e^{c}$$

$$F(x) = \text{Const } X^{\alpha}$$

(iii) Given $F(x+y) = F(x) + F(y)$, differentiating w.r.t y

$$\frac{dF(x+y)}{dx} \cdot \frac{d(x+y)}{dy} = \frac{dF(y)}{dy},$$

$$\frac{dF(x+y)}{dx} = F'(y), \quad \text{Let } y = 0 \text{ and } F'(0) = \text{Const}.$$

$$\frac{d}{dx} F(x) = \text{Const}$$

$$\int d\, F(x) = \int \text{Const } dx$$

$$F(x) = \text{Const } x + c, \quad \text{where } c = 0$$

$$\therefore F(x) = \text{Const } X.$$

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9a. Let us assume that the trade volume between the two countries is described by the gravity model, with $\alpha = 2$.

- How will the trade volume change if the GDP of the first country increases by 50%?
- How will the trade volume change if the GDP of the second country increases by 10%?
- How will the trade volume be different if countries with the same GDP were located at a twice larger distance than the given pair?

9b. What properties were used to justify the gravity model?

9(a): $t = \dfrac{c\, g_1 g_2}{r^2}$ Given

(i) Let $g_1' = 1.5 g_1$ ie 50% increase in g_1

$g_2' = g_2$ and $r' = r$

$\therefore\ t' = \dfrac{c\, g_1' g_2'}{r'} = \dfrac{c(1.5g_1)g_2}{r^2} = 1.5\left(\dfrac{c g_1 g_2}{r^2}\right) = 1.5t$

ie trade volume will increase by 50%

(ii) $t' = \dfrac{c\,(g_1 \cdot g_2')}{r^2}$ where $g_2' = 1.1 g_2$

$= \dfrac{c\,(g_1(1.1 g_2))}{r^2} = 1.1\,\dfrac{c g_1 g_2}{r^2} = 1.1t$

$\therefore$ trade volume will increase by 10%

(iii) If distance increase twice larger then $r' = (2r)^2$

$\therefore\ t' = \dfrac{c\, g_1 g_2}{(2r)^2} = \dfrac{1}{2}\,\dfrac{c g_1 g_2}{r} = \cancel{\frac{1}{2}t}\ \frac{1}{4}$

$\therefore$ Trade volume will reduce by $\left(\frac{1}{2}\right)$ ie 50%

9 (b): The properties to justify the gravity model are

(i) GDP between countries is additive

(ii) distance between countries (economy) is scale invariant.

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9b. What properties were used to justify the gravity model?

9a) =

$\alpha = 2$

$$t = c \cdot \frac{g \cdot g'}{r^{\alpha}}$$

$GDP_1 = 1.5\, GDP_1$

Assume same

$$\bullet\ t = c \cdot \frac{1.5(1)}{r^{\alpha}}$$

Assume same

Increase

$\boxed{\text{it will } \cancel{\text{increase}}\ \cancel{\text{decrease}} \text{ by } 50\%}$

Assume same

$$\bullet\ t = c \cdot \frac{(1)(1.10)}{r^{\alpha}}$$

$GDP_2 = 1.10\,(GDP_2)$

Assume same

$\boxed{\text{it will increase by } 10\%}$

Assume same GDP

$$\bullet\ t = c \cdot \frac{(1)(1)}{(2)^2} = \frac{1}{4} \cdot c \cdot (1)(1)$$

twice longer distance

$\boxed{\text{It will be reduced by a factor of } 4}$

9b) Scale invariant, additive properties

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10. Use the Least Squares method to find the values a and b for which a * x_i + b ~ y_i, based on the following observations:

- $x_1 = -1$, $y_1 = -1$;
- $x_2 = 0$, $y_2 = -1$;
- $x_3 = 1$, $y_3 = 1$.

$$a = \frac{Cov(x, y)}{Var(x)} = \frac{\overline{xy} - \bar{x}\bar{y}}{\overline{x^2} - \bar{x}^2}$$

$$b = \bar{y} - a\bar{x}$$

$$\bar{x} = \frac{-1+0+1}{3} = 0 \qquad \bar{x}^2 = 0$$

$$\bar{y} = \frac{-1-1+1}{3} = -\frac{1}{3} \qquad \bar{x}\bar{y} = 0$$

$$\overline{xy} = \frac{(-1 \cdot -1) + (0 \cdot -1) + (1 \cdot 1)}{3} = \frac{2}{3}$$

$$\overline{x^2} = \frac{(-1)^2 + (0)^2 + (1)^2}{3} = \frac{2}{3}$$

$$\therefore \quad a = \frac{(2/3) - 0}{(2/3) - 0} = \underline{1}$$

$$b = \bar{y} - a\bar{x} = -\frac{1}{3} - (1 \times 0)$$

$$= -\frac{1}{3}$$