# CS 5354/CS 4365 Advanced Computational Methods in Economics and Finance
# Fall 2018, Test 3

Name: ___ ______

100/100

General comments:

- you are allowed up to 5 pages of handwritten notes;
- if you need extra pages, place your name on each extra page;
- the main goal of most questions is to show that you know the corresponding algorithms; so, if you are running of time, just follow the few first steps of the corresponding algorithm;

Good luck!

10/10

1. Use Lagrange multiplier method to solve the following constraint optimization problem: find the point of the line $2x_1 - x_2 = 1$ which is the closest to 0, i.e., in precise terms, minimize the sum $x_1^2 + x_2^2$ under the above constraint.

$f(x) = x_1^2 + x_2^2 \to min$

$g(x) = 2x_1 - x_2 - 1$

$f(x) + g(x) = x_1^2 + x_2^2 + \lambda(2x_1 - x_2 - 1) = 0$

$\frac{\partial}{\partial x_1}: 2x_1 + 2\lambda = 0 \Rightarrow x_1 = -\lambda \Rightarrow x_1 = 2/5$

$\frac{\partial}{x_2}: 2x_2 - \lambda = 0 \Rightarrow x_2 = \frac{\lambda}{2} \Rightarrow x_2 = -1/5$

$2(-\lambda) - (\frac{\lambda}{2}) = 1$

$-2\lambda - \frac{1}{2}\lambda = 1$

$-\frac{5}{2}\lambda = 1$

$\lambda = -2/5$

2-3.

10/10

2. Suppose that we have two investments, one with expected return 10 and variance 20, another with expected return 20 and variance 10, and we want to have a return of 13. Assuming that these two investments are independent, use the general formulas that we had in class to find the optimal portfolio.

3. Same as in Problem 2, but this time, the two investments are not independent: the covariance is -0.5. Describe the optimal portfolio for this case.

10/10

$\mu_1 = 10 \quad \mu_2 = 20 \quad \mu = 13$

$\sigma_1^2 = 20 \quad \sigma_2^2 = 10$

$$\Sigma_0 = \frac{10^0}{20} + \frac{20^0}{10} = \frac{3}{20}$$

2) $$\Sigma_1 = \frac{10^1}{20} + \frac{20^1}{10} = \frac{5}{2}$$

$$\Sigma_2 = \frac{10^2}{20} + \frac{20^2}{10} = 45$$

$$a = \frac{5/2 - 13 \cdot 3/20}{(5/2)^2 - 3/20 \cdot 45} = -1.1$$

$$b = \frac{13 \cdot 5/2 - 45}{(5/2)^2 - 3/20 \cdot 45} = 25$$

$$w_1 = -1.1\left(\frac{10}{20}\right) + \frac{25}{20} = 0.7$$

$$w_2 = -1.1\left(\frac{20}{10}\right) + \frac{25}{10} = 0.3$$

3) Because there are only 2 investments the covariance between them is irrelevant.

$\mu_1 w_1 + \mu_2 w_2 = \mu \quad \Rightarrow \quad 10 w_1 + 20 w_2 - 13 = 0$

$w_1 + w_2 = 1 \qquad\qquad w_1 + w_2 - 1 = 0$

$10 w_2 - 3 = 0$

$w_2 = 0.3, \quad w_1 = 0.7$

4-7. Assume that we have ten estimates for the a company's worth: three estimate of 2 Billion dollars, five estimates of 3 Billions, and two outliers: an over-pessimistic estimate of 0 Billion, and an over-optimistic estimate of 10 Billions.

4. What will be the combined estimate if we use the standard least squares methods (i.e., $l^2$). 10/10

5. What will be the combined estimate if we use the $l^1$ method? Explain in what sense this method is more robust. 10/10

6. What is the general class of robust techniques that includes both $l^2$ and $l^1$ as particular cases? What is the justification for using methods from this class? 10/10

7. Describe the first few steps of an algorithm for providing the $l^p$-estimate for p = 1.5. (No need to actually perform the computations). 10/10

4) $l^2 = a = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{(3\cdot 2)+(5\cdot 3)+(0\cdot 1)+(1\cdot 10)}{3+5+1+1} = 3.1$

5) $l^1 \Rightarrow$ median $\{0, 2, 2, 2, 3, 3, 3, 3, 3, 10\} = 3$

$l^1$ is more robust in the sense that it is less vunerable to outliers.

6) $l^2$ and $l^1$ belong to the general class of $l^p$ methods.
$l^p$ methods are justfied by scale-invariance.

7) set all weights $w_1 \to w_n = 1$ and $p = 1.5$ and some $\varepsilon <<$
while $|a_n - a_{n-1}| \le \varepsilon$ and $|b_n - b_{n-1}| \le \varepsilon$ find a, b such that
$\sum (y_i - (a x_i + b))^2 w_i \to \min$,
We obtain $a = \frac{\overline{xy}\cdot T - \bar{x}\cdot\bar{y}}{\overline{x^2}\cdot T - (\bar{x})^2}$ and $b = \frac{\bar{y} - a\cdot\bar{x}}{T}$ where

$\bar{x} = \sum w_i x_i \quad \overline{xy} = \sum w_i x_i y_i \quad T = \sum w_i$

$\bar{y} = \sum w_i y_i \quad \overline{x^2} = \sum w_i x_i^2$

compute new weights $|w_i| = |y_i - (a x_i + b)|^{2-p}$
recompute a, b such that $\sum (y_i - (a x_i + b))^2 w_i \to \min$ with the new weights

8-10. An investor placed her money into two hedge funds. The first one led to annual returns of 10%, 5%, 5%, 5%, and 10%. The second one lead to annual returns of 9%, 0%, 9%, 9%, and 9%.

8. Which of the two investments leads to better end results? 10/10

9. Which of the two investments will the investor prefer if he/she follows the peak-end rule? 10/10

10. What is the justification for the peak-end rule? 10/10

A: .1, .05, .05, .05, .1 $\Rightarrow$ $1.1 \cdot 1.05 \cdot 1.05 \cdot 1.05 \cdot 1.1 = 1.40$

B: .09, 0, .09, .09, .09 $\Rightarrow$ $1.09 \cdot 1.0 \cdot 1.09 \cdot 1.09 \cdot 1.09 = 1.41$

8) B leads to better results

| | A | B |
|---|---|---|
| first | .1 | .09 |
| last | .1 | .09 |
| best | .1 | .09 |
| worst | .05 | 0 |

9) Following peak-end-rule, the investor will prefer A

10) Peak-end-rule is justified because it is indempotent, associative, scale-invariant, and shift-invariant.