CS 5354/CS 4365 Advanced Computational Methods in Economics and Finance
Fall 2018, Test 3

Name: __

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General comments:

- you are allowed up to 5 pages of handwritten notes;
- if you need extra pages, place your name on each extra page;
- the main goal of most questions is to show that you know the corresponding algorithms; so, if you are running of time, just follow the few first steps of the corresponding algorithm;

Good luck!

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1. Use Lagrange multiplier method to solve the following constraint optimization problem: find the point of the line $2x_1 - x_2 = 1$ which is the closest to 0, i.e., in precise terms, minimize the sum $x_1^2 + x_2^2$ under the above constraint.

$x_1^2 + x_2^2 \to \min$ subject to $2x_1 - x_2 = 1$

Let

$L = x_1^2 + x_2^2 + \lambda(2x_1 - x_2 - 1) \to \min_{x_1, x_2}$

$\frac{\partial L}{\partial x_1}: 2x_1 + 2\lambda = 0 \to x_1 = -\frac{2\lambda}{2} = -\lambda$

$\frac{\partial L}{\partial x_2}: 2x_2 - \lambda = 0 \to x_2 = \frac{\lambda}{2}$

From: $2x_1 - x_2 = 1$

$2(-\lambda) - \frac{\lambda}{2} = 1$

$-4\lambda - \lambda = 2$

$-5\lambda = 2$

$\lambda = -2/5$

$\therefore x_1 = \frac{2}{5}$

$x_2 = \frac{-2/5}{2} = -\frac{2}{10} = -\frac{1}{5}$

check

$2\left(\frac{2}{5}\right) + \frac{1}{5} = \frac{4}{5} + \frac{1}{5} = 1$

2-3.

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2. Suppose that we have two investments, one with expected return 10 and variance 20, another with expected return 20 and variance 10, and we want to have a return of 13. Assuming that these two investments are independent, use the general formulas that we had in class to find the optimal portfolio.

3. Same as in Problem 2, but this time, the two investments are not independent: the covariance is -0.5. Describe the optimal portfolio for this case.

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2) Let $\mu_1 = 10 \quad \sigma_1^2 = 20$ and $\mu_0 = 13$

$\mu_2 = 20 \quad \sigma_2^2 = 10$

$$\Sigma_0 = \Sigma \, 1/\sigma_i^2 = \frac{1}{20} + \frac{1}{10} = 0.15$$

$$\Sigma_1 = \Sigma \, \mu_i/\sigma_i^2 = \frac{10}{20} + \frac{20}{10} = 2.5$$

$$\Sigma_2 = \Sigma \, \mu_i^2/\sigma_i^2 = \frac{10^2}{20} + \frac{20^2}{10} = 45$$

$$a = \frac{\Sigma_1 - \mu_0 \Sigma_0}{\Sigma_1^2 - \Sigma_0 \Sigma_2} = \frac{2.5 - (13 \times 0.15)}{(2.5)^2 - (0.15 \times 45)} = -1.1$$

$$b = \frac{(1 - a\Sigma_1)}{\Sigma_0} = \frac{(1 + [1.1 \times 2.5])}{0.15} = 25$$

$$w_1 = (-1.1)\left(\frac{10}{20}\right) + (25)\left(\frac{1}{20}\right) = 0.7 \text{ to invest. I}$$

$$w_2 = 1 - w_1 = 1 - 0.7 = 0.3 \text{ to invest II}$$

Given $\mu_1 = 10$ $\quad \sigma_1^2 = 20$

$\mu_2 = 20$ $\quad \sigma_2^2 = 10$ $\quad C_{12} = -0.5$

Since we have two investment, the weights are not affected by the covariance, hence

from : $\sum w_i = 1$: $w_1 + w_2 = 1$, — (1)

from : $\sum w_i \mu_i = \mu_0$ $\quad 10w_1 + 20w_2 = 13$ — (2)

Equ (1) $\times 10$ $\quad 10w_1 + 10w_2 = 10$ — (3)

Equ (2) - (3) $\quad 0 + 10w_2 = 3$

$w_2 = 3/10 = 0.3$

$w_1 = 1 - w_2 = 0.7$

4-7. Assume that we have ten estimates for the a company's worth: three estimate of 2 Billion dollars, five estimates of 3 Billions, and two outliers: an over-pessimistic estimate of 0 Billion, and an over-optimistic estimate of 10 Billions.

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4. What will be the combined estimate if we use the standard least squares methods (i.e., l^2).

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5. What will be the combined estimate if we use the l^1 method? Explain in what sense this method is more robust.

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6. What is the general class of robust techniques that includes both l^2 and l^1 as particular cases? What is the justification for using methods from this class?

7. Describe the first few steps of an algorithm for providing the l^p-estimate for p = 1.5. (No need to actually perform the computations).

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estimates are 2, 2, 2, 3, 3, 3., 3., 3, 0, 10

(4) Combined estimate by $L^2 = \frac{3(2)+5(3)+0+10}{10} = 3.1$

ie Arithmetic average.

(5) (a) Combined estimate by L^1 is the median given by

0 2 2 2 3 | 3 3 3 3 10

$\text{median} = \frac{3+3}{2} = 3$

(b) l^1 is more robust because the Combined estimate is not affected by the outliers 0, 10

(6) (a) General class that include L^1 and l^2 is L^p method.

(b) Justification is that, the estimate $L(e)$ is scale invariant with respectet to the error (e)

ie $L(e) = \text{Cost}\,|e|^p$

L^p - Algorithm with $p = 1.5$

idea: $\min\limits_{\beta} \sum\limits_{k=1}^{n} |(y_k - f(x_k, \beta))|^{1.5} = \min\limits_{\beta} \sum\limits_{k=1}^{n} \frac{|(y_k - f(x_k,\beta))|^2}{|(y_k - f(x_k,\beta))|^{0.5}}$

where β is parameter to be estimated

Given x_k, y_k for $k = 1, \ldots n$, we want to find β for which

$$\sum_{k=1}^{n} |y_k - f(x_k, \beta)|^{1.5} \longrightarrow \min.$$

step 1: we use least squares method to find $\beta^{(1)}$ for which $\sum\limits_{k=1}^{n} |y_k - f(x_k, \beta)|^2 \longrightarrow \min$

Step s, $s = 2, 3, \ldots$ we take $\beta^{(s-1)}$ and Compute

$|e_k^{(s)}| = |y_k - f(x_k, \beta^{(s-1)})|$ and find β

for which $\sum\limits_{k=1}^{n} \frac{(y_k - f(x_k, \beta))^2}{|e_k^{(s)}|^{0.5}} \longrightarrow \min$

this is $\beta^{(s)}$

Next we continue with steps until $|\beta^{(s)} - \beta^{(s-1)}| \le \varepsilon$ ie the required accuracy is obtained.

8-10. An investor placed her money into two hedge funds. The first one led to annual returns of 10%, 5%, 5%, 5%, and 10%. The second one lead to annual returns of 9%, 0%, 9%, 9%, and 9%.

8. Which of the two investments leads to better end results? 10/10

9. Which of the two investments will the investor prefer if he/she follows the peak-end rule? 10/10

10. What is the justification for the peak-end rule? 10/10

(8) Invest I total return = 1.1 × 1.05 × 1.05 × 1.05 × 1.1 = 1.4007

Invest. II total return = 1.09 × 1 × 1.09 × 1.09 × 1.09 = 1.4116

In reality invest. II is better.

(9) Peak - end rule

	Invst I	Invest II
First	10	9
Last	10	9
minimum	5	0
maximum	10	9

Comparing I and II, by peak end rule I is better

(10) Justification of peak - end rule is that, the Combined Satisfaction or experience

(1) Assoziative
(2) idempotent
(3) Scale invariant
(4) Shift invariant.