CS 5354/CS 4365 Intelligent Computing
Fall 2019, Test 1

Name: _____

$\frac{70}{70} = \left(\frac{100}{100}\right)$

General comments:

- you are allowed up to 5 pages of handwritten notes;
- ~~if you need extra pages, place the last 4 digits of ID number on each extra page;~~
- the main goal of most questions is to show that you know the corresponding algorithms; so, if you are running of time, just follow the few first steps of the corresponding algorithm;
- each question will be graded on its own merit; so, for example, if when answering to the first part of the question, you got the wrong formula, but on the second part, you correctly used this formula, you will get full credit for the second part.
- if you need extra pages, add your name to each additional page.

Good luck!

1. Explain why we need fuzzy control methodology in the first place. 10/10

To formalize formal knowledge

In many application areas, there are experts whose experience we would like to capture. Many times the rules they describe use imprecise "fuzzy" words from natural language like "big", "small". This makes it difficult to describe to a computer, fuzzy logic helps in this

12/19

2. If the degree of confidence in a statement A is 0.8 and the degree of confidence in a statement B is 0.4, then what are the estimated degrees of confidence in statements A & B and A \/ B? Consider two cases:

1,2 • "and"-operation is min(a,b) and "or"-operation is a + b − a*b;
3,4 • "and"-operation is a*b and "or"-operation is min(a + b, 1).

A is 0.8

B is 0.4

① Using $\min(A,B)$ for "and" operation

$A \wedge B = \min(0.8, 0.4) = \boxed{0.4}$

② Using $a+b-a \cdot b$ for "or" operation

$A \vee B = A + B - A \cdot B = 0.8 + 0.4 - 0.8 \cdot 0.4 = 1.2 - 0.32 = \boxed{0.88}$

③ "And" operation using $A \cdot B$

$A \wedge B = A \cdot B = 0.8 \cdot 0.4 = \boxed{0.32}$

④ "Or"-operation using $\min(A+B, 1)$

$A \vee B = \min(A+B, 1) = \min(0.8+0.4, 1) = \min(1.2, 1) = \boxed{1}$

10/10

3. Let us consider the following expert rules:

- if a student studies a lot, this student gets a good grade;
- if a student studies little, this student gets a not very good grade.

Describe step-by-step -- like we did in class and like it is described in the handout -- how you would translate these rules into a formula for the corresponding predicate R(t, g) meaning that if the students spends time t on studying, then it is reasonable to expect a grade g.

Let us now assume that the student studied for 6 hours, for which:

- the degree to which this time is little is 0.3, and
- the degree to which this is a lot is 0.7.

Let us assume that we use max(a + b − 1, 0) as an "and"-operation and min(a + b, 1) as an "or"-operation. What is the degree to which it is reasonable to expect that this student will get a grade of 80 if:

- the degree to which this grade is good is 0.8, and
- the degree to which this grade is not very good is 0.2.

LT(t) will indicate that time t is little
LOTT(t) will indicate that time t is a lot
G(g) indicates that the grade g is good
B(g) indicates that the grade g is not very good

R(t,g)

LOTT(t) → G(g);
LT(t) → B(g)

R(t,g) = (LOTT(t) & G(g)) ||
(LT(t) & B(g))

$R_k(t,g)$ will indicate that k^{th} rule is applicable for given t, and that this recommends to use value g.

C(t,g) will indicate that g is a reasonable for given input t

$C(t,g) \equiv R_1(t,g) \vee R_2(t,g)$

where
$R_1(t,g) \equiv LOTT(t) \,\&\, G(g)$;
$R_2(t,g) \equiv LT(t) \,\&\, B(g)$

We can define degree of belief $d_i(t,g)$ that i-th rule will be fired

$$d_1(t,g) = f_{\&}(\mu_{LOTT}(t), \mu_G(g))$$
$$d_2(t,g) = f_{\&}(\mu_{LT}(t), \mu_B(g))$$

$$\mu_c(t,g) = f_{\vee}(d_1(t,g), d_2(t,g))$$

And operation is $\max(a+b-1, 0)$, Or operation is $\min(a+b, 1)$

$t=6$, $g=80$

$\mu_{LOTT}(t) = 0.7$

$\mu_{LT}(t) = 0.3$

$\mu_G(g) = 0.8$

$\mu_B(g) = 0.2$

$$d_1(t,g) = \max(\mu_{LOTT}(t) + \mu_G(g) - 1, 0)$$
$$= \max(0.7 + 0.8 - 1, 0) = 0.5$$

$$d_2(t,g) = \max(\mu_{LT}(t) + \mu_B(g) - 1, 0)$$
$$= \max(0.3 + 0.2 - 1, 0) = 0$$

$$\mu_c(t,g) = \min(d_1(t,g) + d_2(t,g), 1)$$
$$= \min(0.5 + 0, 1) = 0.5$$

$$\boxed{\mu_c(t,g) = 0.5}$$ for $t=6$ $g=80$

10/10

4. Use calculus to find the values x and y for which the value $x^2 - x*y + y^2 + x + 2y$ is the smallest possible.

$f(x,y) = x^2 - x \cdot y + y^2 + x + 2y$

$\frac{\partial f}{\partial x} = 2x - y + 0 + 1 + 0 = 2x - y + 1$

$\frac{\partial f}{\partial y} = 0 - x + 2y + 0 + 2 = -x + 2y + 2$

$2x - y + 1 = 0$

$-x + 2y + 2 = 0$

$x = 2y + 2$

$2 \cdot (2y+2) - y + 1 = 0$

$4y + 4 - y + 1 = 0$

$3y + 5 = 0$

$y = -\frac{5}{3}$

$2x - y + 1 = 0$

$2x - (-\frac{5}{3}) + 1 = 0$

$2x + \frac{8}{3} = 0$

$2x = -\frac{8}{3}$

$x = -\frac{4}{3}$

Minimum at $x = -\frac{4}{3}$ and $y = -\frac{5}{3}$

10/10

5. Suppose that a function f(x) of a real variable x is already implemented in software. Use the resulting method that inputs the values a and b and returns the integral of f(x) on the interval [a, b].

$$\int_a^b f(x)\,dx = \sum f(x_i)\Delta x = (\Sigma f(x)) \cdot \Delta x$$

Good approximation as Δx approaches 0

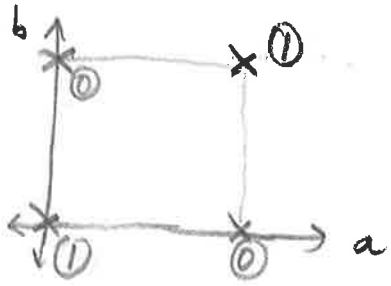
```
double integral(double a, double b){
    double x = a;
    double delta_x = 0.0001;
    sum = 0;
    while (x <= b){
        sum += f(x) * delta_x;
        x += delta_x;
    }

    return sum;
}
```

10/10

6. Use linear interpolation to come up with a fuzzy version of the equivalent relation. For Boolean variable, the equivalence is eq(a, b) = 1 if a = b, else eq(a, b) = 0.

$f(x) = y_1 + \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$



1 Lets fix $a = 0$

$f_1(b) = f_v(0, b)$

$f_1(0) = f_v(0, 0) = 1$

$f_2(1) = f_v(0, 1) = 0$

$x_1 = 0 \rightarrow y_1 = 1$

$x_2 = 1 \rightarrow y_2 = 0$

$f_1(x) = 1 + \frac{0-1}{1-0}(x - 0) = 1 + -x = 1 - x$

$f_v(0, b) = 1 - b$

2 Lets fix $a = 1$

$f_2(b) = f_v(1, b)$

$f_2(0) = f_v(1, 0) = 0$

$f_2(1) = f_v(1, 1) = 1$

$f_2(x) = x$

$f_v(1, b) = b$

3 Lets fix b

$f_3(a) = f_v(a, b)$

$f_3(0) = f_v(0, b) = 1 - b$

$f_3(1) = f_v(1, b) = b$

$x_1 = 0 \rightarrow y_1 = 1 - b$

$x_2 = 1 \rightarrow y_2 = b$

$f_3(x) = (1 - b) + \frac{b - (1 - b)}{1 - 0}(x - 0)$

$= 1 - b + (2b - 1)x$

$= 1 - b + 2bx - x$

$f_v(a, b) = 1 - b + 2ba - a$