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6. Solve all parts of this problem.

6.a. What is the result of applying Hadamard transform to the state $0.6|0\rangle - 0.8|1\rangle$?

6.b. What is the result of applying the quantum version of negation to the state $0.6|0\rangle - 0.8|1\rangle$?

6.c. What is the result of applying the quantum version of "or"-operation to the states $0.6|0\rangle - 0.8|1\rangle$ and $0.8i|0\rangle - 0.6|1\rangle$?

Q6a.

$$H(0.6|0\rangle - 0.8|1\rangle) = H(0.6|0\rangle) - H(0.8|1\rangle)$$

$$H(0.6|0\rangle) = 0.6\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) = \frac{3}{5\sqrt{2}}|0\rangle + \frac{3}{5\sqrt{2}}|1\rangle$$

$$H(0.8|1\rangle) = 0.8\left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right) = \frac{4}{5\sqrt{2}}|0\rangle - \frac{4}{5\sqrt{2}}|1\rangle$$

Thus

$$H(0.6|0\rangle - 0.8|1\rangle) = \frac{3}{5\sqrt{2}}|0\rangle + \frac{3}{5\sqrt{2}}|1\rangle - \frac{4}{5\sqrt{2}}|0\rangle + \frac{4}{5\sqrt{2}}|1\rangle$$

$$= \frac{-1}{5\sqrt{2}}|0\rangle + \frac{7}{5\sqrt{2}}|1\rangle$$

Q6b.

$$(0.6|0\rangle - 0.8|1\rangle) \otimes |0\rangle$$

$$0.6|00\rangle - 0.8|10\rangle$$

Now we apply $f(x) = 7x$

$|x, y\rangle \rightarrow |x, y \oplus f(x)\rangle$

$|00\rangle \rightarrow |01\rangle$

$|10\rangle \rightarrow |10\rangle$

Thus we have

$$0.6|01\rangle - 0.8|10\rangle$$

Q6c.

$$(0.6|0\rangle - 0.8|1\rangle) \otimes (0.8i|0\rangle - 0.6|1\rangle) \otimes |1\rangle$$

$$= 0.48i|001\rangle - 0.36|011\rangle - 0.64i|101\rangle + 0.48|111\rangle$$

$f(x) = x_1$ or x_2

$|x_1, x_2, y\rangle \longrightarrow |x_1, x_2, y \oplus f(x)\rangle$

$|001\rangle \longrightarrow |001\rangle$

$|011\rangle \longrightarrow |010\rangle$

$|101\rangle \longrightarrow |100\rangle$

$|111\rangle \longrightarrow |110\rangle$

Thus the final result is

$$0.48i\,|001\rangle - 0.36\,|010\rangle - 0.64i\,|100\rangle + 0.48\,|110\rangle$$

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8. Solve only one part of this problem -- corresponding to your UTEP ID.

8.0 What will happen if we apply the function f(x) = 1 to a general 2-bit state

$$a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle?$$

8.1. What will happen if we apply the function f(x) = not x to a general 2-bit state

$$a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle?$$

8.2. What will happen if we apply the function f(x) = x to a general 2-bit state

$$a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle?$$

8.3. What will happen if we apply the function f(x) = 0 to a general 2-bit state

$$a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle?$$

8.3 $|x,y\rangle \rightarrow |x, y \oplus 0\rangle$

$a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$

8.0 $|x,y\rangle \rightarrow |x, y \oplus 1\rangle$

$a|01\rangle + b|00\rangle + c|11\rangle + d|10\rangle$

8.1 $|x,y\rangle \rightarrow |x, y \oplus \neg x\rangle$

$a|01\rangle + b|00\rangle + c|10\rangle + d|11\rangle$

8.2 $|x,y\rangle \rightarrow |x, y \oplus x\rangle$

$a|00\rangle + b|01\rangle + c|11\rangle + d|10\rangle$

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8. Solve only one part of this problem -- corresponding to your UTEP ID.

8.0 What will happen if we apply the function f(x) = 1 to a general 2-bit state

$$a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle?$$

8.1. What will happen if we apply the function f(x) = not x to a general 2-bit state

$$a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle?$$

8.2. What will happen if we apply the function f(x) = x to a general 2-bit state

$$a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle?$$

8.3. What will happen if we apply the function f(x) = 0 to a general 2-bit state

$$a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle?$$

$|00\rangle \rightarrow |1, 0 \oplus 0\rangle = |1\,0\rangle$

$|01\rangle \rightarrow |1, 1 \oplus 0\rangle = |1\,1\rangle$

$|10\rangle \rightarrow |1, 0 \oplus 1\rangle = |1\,1\rangle$

$|11\rangle \rightarrow |1, 1 \oplus 1\rangle = |1\,0\rangle$

the result is $a|00\rangle + b(01\rangle + c|11\rangle + d|10\rangle$

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8. Solve only one part of this problem -- corresponding to your UTEP ID.

8.0 What will happen if we apply the function f(x) = 1 to a general 2-bit state

$$a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle?$$

8.1. What will happen if we apply the function f(x) = not x to a general 2-bit state

$$a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle?$$

8.2. What will happen if we apply the function f(x) = x to a general 2-bit state

$$a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle?$$

8.3. What will happen if we apply the function f(x) = 0 to a general 2-bit state

$$a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle?$$

8.3) if we apply $f(x)=0$ to $a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$ we get the same state since $y \oplus 0 = y$