

Solution to Homework 15

Problem. As you know, in the usual 2-valued logic, negation is defined by the formulas $f(0) = 1$ and $f(1) = 0$. We would like to extend this function $f(x)$ to all possible values x from the interval $[0, 1]$. Such an extension is known as the *negation operation*. What is the most robust negation operation?

Solution. What we want is an example of interpolation. We want to find the function $f(x)$ for which:

- for $x_1 = 0$ we have $y_1 = f(x_1) = 1$, and
- for $x_2 = 1$ we have $y_2 = f(x_2) = 0$.

We have proven that the most robust interpolation is linear interpolation. By applying the linear interpolation formula, we get

$$f(x) = y_1 + \frac{y_2 - y_1}{x_2 - x_1} \cdot (x - x_1) = 1 + \frac{0 - 1}{1 - 0} \cdot (x - 0) = 1 - x.$$