Solution to Homework 17

Task. Use the least squares method to find the dependence $y = c_1 \cdot x + c_2$ for the case when we have the following three measurements:

- $x^{(1)} = -2$, $y^{(1)} = 1$;
- $x^{(2)} = 0$, $y^{(2)} = -1$;
- $x^{(3)} = 2$, $y^{(3)} = -1$.

Solution. Here, $K = 3$,

$$\overline{x} = x^{(1)} + x^{(2)} + x^{(3)} = (-2) + 0 + 2 = 0,$$

$$\overline{y} = y^{(1)} + y^{(2)} + y^{(3)} = 1 + (-1) + (-1) = -1;$$

$$\overline{x^2} = \left(x^{(1)}\right)^2 + \left(x^{(2)}\right)^2 + \left(x^{(3)}\right)^2 = (-2)^2 + 0^2 + 2^2 = 4 + 0 + 4 = 8;$$

$$\overline{x \cdot y} = x^{(1)} \cdot y^{(1)} + x^{(2)} \cdot y^{(2)} + x^{(3)} \cdot y^{(3)} =$$

$$(-2) \cdot 1 + 0 \cdot (-1) + 2 \cdot (-1) = -2 + 0 + (-2) = -4.$$

So,

$$c_1 = \frac{K \cdot \overline{x \cdot y} - \overline{x} \cdot \overline{y}}{K \cdot \overline{x^2} - \left(\overline{x}\right)^2} = \frac{3 \cdot (-4) - 0 \cdot (-1)}{3 \cdot 8 - 0^2} = \frac{-12}{24} = -\frac{1}{2} = -0.5$$

and

$$c_2 = \frac{\overline{x^2} \cdot \overline{y} - \overline{x} \cdot \overline{x \cdot y}}{K \cdot \overline{x^2} - \left(\overline{x}\right)^2} = \frac{8 \cdot (-1) - 0 \cdot (-4)}{3 \cdot 8 - 0^2} = \frac{-8}{24} = -\frac{1}{3} = -0.33\ldots$$