Solution to Homework 18

Task. Suppose that we know that $f(0) = 2$ and $f(2) = 1$, and we want to find the value $f(1)$ that minimizes the following expression

$$(f(1) - f(0))^2 + (f(2) - f(1))^2.$$

Use variational derivative to find this value.

Answer. Differentiating with respect to $f(1)$, we get

$$2 \cdot (f(1) - f(0)) \cdot (-1) + 2 \cdot (f(2) - f(1)) \cdot 1 = 0.$$

Dividing both sides by 2, we get:

$$(f(1) - f(0)) \cdot (-1) + (f(2) - f(1)) = 0,$$

so

$$f(1) - f(0) + f(2) - f(1) = 2f(1) - f(0) - f(2) = 0.$$

To separate the variable, we add $f(0)$ and $f(2)$ to both sides, getting

$$2f(1) = f(0) + f(2),$$

so

$$f(1) = \frac{f(0) + f(2)}{2}.$$

In our case, $f(0) = 2$ and $f(2) = 1$, so

$$f(1) = \frac{2 + 1}{2} = \frac{3}{2} = 1.5.$$