Solution to Problem 20

Task. We have shown that the only maximally individually robust "or"-operation is $\max(a, b)$. Maximally individually robust means, in this case, that for all possible values a, b, a', and b', we must have

$$|f_\vee(a, b) - f_\vee(a', b')| \le \max(|a - a'|, |b - b'|).$$

Provide an example of the values a, b, a', and b', showing that the "or"-operation $a+b-a\cdot b$ is not maximally individually robust. *Hint:* it is sufficient to consider values 0, 0.5, and 1.

Solution. For $a = b = 0$ and $a' = b' = 0.5$, we have $f_\vee(a, b) = f_\vee(0, 0) = 0$ and

$$f_\vee(a', b') - f_\vee(0.5, 0.5) = 0.5_0.5 - 0.5 \cdot 0.5 = 1 - 0.25 = 0.75.$$

So here $|f_\vee(a, b) = f_\vee(a', b')| = |0 - 0.75| = 0.75$.

On the other hand, here, $|a - a'| = |b - b'| = |0 - 0.5| = 0.5$, so

$$\max(|a - a'|, |b - b'|) = \max(0.5, 0.5) = 0.5.$$

Thus, here

$$|f_\vee(a, b) - f_\vee(a', b')| > \max(|a - a'|, |b - b'|).$$