Solution to Homework 2

General background. We know that:

- for $x_1 = 1$, we have $y_1 = 2$, and
- for $x_2 = 2$, we have $y_2 = 3$.

First task. Use the general linear interpolation formula that we had in class to come up with the expression $y = f(x)$ for the dependence of y on x.

Solution.

$$f(x) = y_1 + \frac{y_2 - y_1}{x_2 - x_1} \cdot (x - x_1) = 2 + \frac{3-2}{2-1} \cdot (x-1) = 2 + x - 1 = x + 1.$$

Second task. For your expression $f(x)$, what is the value of $f(1.5)$?

Solution. $f(1.5) = 1.5 + 1 = 2.5$.

Third task. How is linear interpolation used in fuzzy techniques?

Solution. For each natural-language property like "negligible", we elicit, from the expert, the degrees $\mu\left(x^{(k)}\right)$ to which several values $x^{(k)}$ satisfy this property (i.e., to which $x^{(k)}$ is negligible). To find the degrees $\mu(x)$ corresponding to all other values x, we need to use interpolation – and linear interpolation is one of these techniques.

Fourth task. Explain, step by step, how we can derive the general formula for the linear interpolation.

Solution. Linear interpolation assumes that the dependence of a quantity y on a quantity x is linear, i.e., that

$$y(x) = a \cdot x + b \tag{1}$$

for some values a and b. We know two cases in which we measured x and y:

- we know the values x_1 and y_1 for which

$$a \cdot x_1 + b = y_1, \tag{2}$$

  and

- we know the values x_2 and y_2 for which

$$a \cdot x_2 + b = y_2. \tag{3}$$

Based on this information, we need to find the formula for $y(x)$.

Subtracting (2) from (3), we conclude that

$$a \cdot (x_2 - x_1) = y_2 - y_1. \tag{4}$$

Dividing both sides by the difference $x_2 - x_1$, we conclude that

$$a = \frac{y_2 - y_1}{x_2 - x_1}. \tag{5}$$

Subtracting (2) from (1), we conclude that

$$a \cdot (x - x_1) = y - y_1, \tag{6}$$

therefore

$$y = y_1 + a \cdot (x - x_1). \tag{7}$$

Substituting the expression (5) for a into this formula, we conclude that

$$y = y_1 + (x - x_1) \cdot \frac{y_2 - y_1}{x_2 - x_1}. \tag{8}$$