

Solution to Homework 4

Task. Let us consider the following expert rules:

- if a cat is somewhat bored and you have a little bit of time, play with it a little bit;
- if a cat is very bored and you have a lot of time, play with it for a long time.

Describe step-by-step – like we did in class and like it is described in the corresponding paper – how you would translate these rules into a formula for the corresponding predicate $R(b, t, p)$ meaning that if the cat is in the state b and you have time t , then it is reasonable to play it for time p .

Let us now assume that:

- the cat is somewhat bored with degree 0.3 and very bored with degree 0.7,
- $t = p = 1$ hour; the degree to which this time is a little bit is 0.4, the degree to which this is a long time is 0.6;
- we use $a \cdot b$ as the “and”-operation and $a + b - a \cdot b$ as the “or”-operation.

What will then be the resulting degree $\mu_R(b, t, p)$?

Solution: general idea. First, we design a statement that describes when the value p is reasonable. This statement means that:

- either the first rule is applicable, i.e., its conditions are satisfied and its conclusion is satisfied,
- or the second rule is applicable, i.e., its conditions are satisfied and its conclusion is satisfied.

Let us design statements corresponding to the rules.

Transforming the first rule into an “and”-statement. In the first rule, we have two conditions:

- a cat is somewhat bored, and
- you have a little bit of time,

and we have one conclusion: “play a little bit”. So, that the first rule is applicable means that the following “and”-statement is satisfied:

cat is somewhat bored *and* you have a little bit of time *and*
you play a little bit.

Transforming the second rule into an “and”-statement. In the second rule, we also have two conditions:

- a cat is very bored,
- you have a lot of time,

and we have one conclusion: “play for a long time”. So, that the second rule is applicable means that the following “and”-statement is satisfied:

cat is very bored *and* you have a lot of time *and* you play for a long time.

Combining the “and”-statements corresponding to the two rules. The play time is reasonable if one of the two rules is applicable, i.e., when the following “or”-statement is true:

(cat is somewhat bored *and* you have a little bit of time *and*
you play a little bit) *or*
(cat is very bored *and* you have a lot of time *and* you play for a long time).

In terms of the variables b , t , and p , we rewrite this “or”-statement statement as follows:

(b is somewhat bored *and* t is a little bit *and* p is a little bit) *or*
(b is very bored *and* t is a lot of time *and* p is a long time).

Computing the degree to which the first rule is applicable. In the part corresponding to the first rule:

- the degree to which the cat is somewhat bored is 0.3,
- the degree to which $t = 1$ is a little bit is 0.4, and
- the degree to which $p = 1$ is a little bit is 0.4.

So, for the degree corresponding to the first rule, we get

$$f_{\&}(0.3, 0.4, 0.4) = 0.3 \cdot 0.4 \cdot 0.4 = 0.12 \cdot 0.4 = 0.048.$$

Computing the degree to which the second rule is applicable. Similarly, for the second rule, we get

$$f_{\&}(0.7, 0.6, 0.6) = 0.7 \cdot 0.6 \cdot 0.6 = 0.42 \cdot 0.6 = 0.252.$$

Final step: computing the degree to which the given play time is reasonable. By applying an “or”-operation to the degrees corresponding to the two rules, we get the desired degree:

$$\mu_R(b, t, p) = f_{\vee}(0.048, 0.252) = 0.048 + 0.252 - 0.048 \cdot 0.252 \approx 0.3 - 0.012 = 0.288.$$