Test 1 for Explainable Fuzzy AI Class
Solutions, Summer 2021

Question 1a. What is explainable AI and why do we need it?

Answer. Many AI programs – in particular, the ones that use deep learning – just provide a recommendation, they do not come with any explanation. We know that these programs are not perfect, that sometimes their recommendations are wrong – but since there are no explanations, we do not know which recommendations are wrong. It is therefore desirable to have such explanations.

When an AI system not only provides *recommendations*, but also provides *explanations* for these recommendations, this is called explainable AI.

Question 1b. What are fuzzy techniques and what is their purpose?

Answer. Fuzzy techniques are techniques for translating expert knowledge which has been formulated by using imprecise ("fuzzy") words from a natural language (like "small") into precise computer-understandable terms.

Question 1c. Who invented fuzzy techniques?

Answer. Lotfi Zadeh.

Question 1d. Why is it reasonable to use fuzzy techniques in explainable AI?

Answer. Desire for explanations means that we need to be able to transform numerical recommendations into natural-language explanations. In other words, we need to connect numerical recommendations with natural-language rules.

Such a connection has been explored before: this is exactly what fuzzy techniques are about.

Question 2a. If an expert marked 3 on a scale from 0 to 4, what is the resulting degree of confidence?

Answer. $3/4 = 0.75$.

Question 2b. If 4 out of 5 experts believe that the statement S is correct, what it its degree of confidence?

Answer. $4/5 = 0.8$.

Question 2c. Why do we need interpolation in fuzzy techniques?

Answer. To describe a natural-language property like "small", fuzzy technique assigns, to each value x of the corresponding quantity, the degree $\mu(x)$ to which this value satisfies the given property – e.g., to which x is small. These degrees should come from the experts. However, there are infinitely many possible values x, but we can only asked finitely many questions. Thus, from the experts, we can only get finitely many values $\mu\left(x^{(1)}\right)$, $\mu\left(x^{(2)}\right)$, ... To get the values $\mu(x)$ for all other x, we need to use interpolation/extrapolation.

Question 2d. What is a membership function?

Answer. For each natural-language expression P (e.g., "small"), a membership function is a function that assigns, to each possible value x of a quantity, the degree $\mu_P(x)$ to which this value satisfies the given property (e.g., to which this value is small).

Question 2e. Assume that $\mu(-2) = 1$ and $\mu(0) = 0$. Use linear interpolation to find $\mu(-1)$.

Answer. Here, for $x_1 = -2$, we have $y_1 = 1$, and for $x_2 = 0$, we have $y_2 = 0$. Thus, for $x = -1$, we have

$$\mu(-1) = y_1 + \frac{y_2 - y_1}{x_2 - x_1} \cdot (x - x_1) = 1 + \frac{0 - 1}{0 - (-2)} \cdot (-1 - (-2)) =$$

$$1 + \frac{-1}{2} \cdot 1 = 1 - \frac{1}{2} = \frac{1}{2} = 0.5.$$

Question 3a. What is an "and"-operation? What is an "or"-operation?

Answer. An "and"-operation is an algorithm that transforms our degrees of confidence a and b in statements A and B into an estimate $f_\&(a,b)$ for our degree of confidence in the statement "A and B" ($A \,\&\, B$).

An "or"-operation is an algorithm that transforms our degrees of confidence a and b in statements A and B into an estimate $f_\vee(a,b)$ for our degree of confidence in the statement "A or B" ($A \vee B$).

Question 3b. Assume that our degree of confidence in A is 0.6, and degree of confidence in B is 0.7. Use min, max, algebraic product, and $a + b - a \cdot b$ to estimate degrees of confidence in $A \,\&\, B$ and $A \vee B$.

Answer. The degree of confidence in $A \,\&\, B$ is equal to either $\min(0.6, 0.7) = 0.6$ or to $0.6 \cdot 0.7 = 0.42$.

The degree of confidence in $A \vee B$ is equal to either $\max(0.6, 0.7) = 0.7$ or to $0.6 + 0.7 - 0.6 \cdot 0.7 = 1.3 - 0.42 = 0.88$.

Problem 4. Suppose that we have two rules:

- if a student studied hard, the student will get a good grade;
- if a student studied very hard, the student will get a very good grade.

A student studied for 3 hours and got 88/100 on the test. Assume that:

- the degree to which 3 hours means studying hard is 0.6, and the degree to which it means studying very hard is 0.4;
- the degree to which 88 is a good grade is 0.8, and the degree to which 88 is a very good grade is 0.2.

Based on this information, what is the degree to which the student's grade is reasonable? Use min and max.

Answer. The grade is reasonable if:

- either the first rule is applicable, i.e., its condition(s) are satisfied, and its conclusion is true,
- or the second rule is applicable, i.e., its condition(s) are satisfied, and its conclusion is true.

In this case, it means that

(student studies hard *and* got a good grade) *or*
(student studied very hard *and* got a very good grade).

Here:

- the degree to which the first rule is applicable is
$$f_\&(0.6, 0.8) = \min(0.6, 0.8) = 0.6;$$
- the degree to which the second rule is applicable is
$$f_\&(0.4, 0.2) = \min(0.4, 0.2) = 0.2.$$

Thus, the degree to which the grade is reasonable is

$$f_\vee(0.6, 0.2) = \max(0.6, 0.2) = 0.6.$$

Question 5a. What is the distance $D(a,b)$ between the points $a = (-2,-3)$ and $b = (1,-7)$?

Answer.

$$D(a,b) = \sqrt{(-2-1)^2 + ((-3)-(-7))^2} = \sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25} = 5.$$

Question 5b. What is the squared distance $D^2(a,b)$ between the points $a = (0,2,4)$ and $b = (0,-2,-4)$?

Answer.

$$D^2(a,b) = (0-0)^2 + (2-(-2))^2 + (4-(-4))^2 = 0^2 + 4^2 + 8^2 = 16 + 64 = 80.$$

Question 5c. Use differentiation to find the minimum (= smallest value) of the expression $x^2 - 2x + 1$.

Answer. At the point where the minimum is attained, the derivative is equal to 0. For this function, the derivative is equal to $2x - 2$. This expression is equal to 0 when $2x - 2 = 0$, then $2x = 2$ and $x = 1$.

For $x = 1$, the value of the function is $1^2 - 2 \cdot 1 + 1 = 1 - 2 + 1 = 0$. So, the desired smallest value is 0. medskip

Question 5d. What is defuzzification and why do we need it?

Answer. By applying fuzzy techniques to the expert rules, for each possible value u of control, we can generate the degree $\mu(u)$ to which this value is reasonable. As a result, we get what is called a membership function (or a fuzzy set) $\mu(u)$.

If we are designing an automatic system, then we need to generate a *single* control value $\overline{u}$ that the system will apply. So, we need to transform the fuzzy set into an exact value. This transformation is known as *defuzzification*.

Question 5e. Suppose that we have the following reasonableness degrees: for $u_1 = -1$, we have $\mu(u_1) = 0.5$, and for $u_2 = 1$, we have $\mu(u_2) = 1$. What will be the result of centroid defuzzification?

Answer.

$$\bar{u} = \frac{u_1 \cdot \mu(u_1) + u_2 \cdot \mu(u_2)}{\mu(u_1) + \mu(u_2)} = \frac{(-1) \cdot 0.5 + 1 \cdot 1}{0.5 + 1} = \frac{0.5}{1.5} = 0.333\ldots$$