Test 2 for Explainable Fuzzy AI Class
Summer 2021

General comment. Test time is limited, so just answer the questions:

- If a question does not explicitly asks to explain why, e.g., if the question is "Which interpolation is the most robust?", just list the most robust interpolation, no need to reproduce how the corresponding formulas were derived.
- Of course, if the question explicitly asks for explanations and/or derivations, please provide these explanations and/or derivations.

Good luck!

1. *Robustness: definitions and results*

1a. What is robustness?

1b. Why do we want membership functions to be robust?

1d. Which interpolation is the most robust?

1c. Why do we want "and"- and "or"-operations to be robust?

1e. Which "and"- and "or"-operations are the most robust?

2. *Robustness: techniques.*

2a. What is variational optimization?

2b. What is variational derivative and why do we need it?

2c-d. Suppose that we know that $f(0) = 1$ and $f(1) = 3$, and we want to find the value $f(0.5)$ that minimizes the following expression

$$(f(0) - f(0.5))^2 + (f(0.5) - f(1))^2.$$

Use variational derivative to find this value.

3. *Individual robustness.*

3a. Suppose that we know the values $f(a)$ and $f(b)$ for some $a < b$. What does it mean for an interpolating function $f(x)$ to be individually robust? Provide a precise definition.

3b. Which interpolation is the most individually robust?

3c. Provide an example showing that when we know that $f(0) = 1$ and $f(1) = 0$, the function $f(x) = (1 - x)^2$ is not maximally individually robust.

3d. Which "and"-operation is the most individually robust?

3e. Provide an example showing that algebraic product is not maximally individually robust.

4. *Least squares.*

- Suppose that we know that $f(0) = 0$, $f(1) = 1$, and $f(2) = 4$.
- Use the least squares formulas to come up with the best linear approximation to this data.

5. *Group vs. individual control.*

5a-b. Which "and" and "or"-operations should we use in the following two situations:

- if we are controlling a group of objects, and malfunctioning of one of them is OK as long as, on average, they all fulfil their mission;
- if we are controlling a single object.

5c. So, how can we use fuzzy techniques in explainable AI?

5d. What is tuning and how is it different from machine learning?