Solution to Homework 2

Task. Suppose that you know the values of some quantity v in three points x_1, x_2, and x_3, these values are $v_1 = 1$, $v_2 = 2$, and $v_3 = 3$. Based on this information, you want to use the inverse distance weighting technique to predict the value v of this quantity as a point x for which $d(x_1, x) = 10$, $d(x, x_2) = 20$, and $d(x, x_3) = 30$. Take $a = -1$.

Solution. The general formula has the form

$$v = \frac{\sum\limits_i v_i \cdot (d(x, x_i))^a}{\sum\limits_i (d(x, x_i))^a}.$$

In our case, the numerator is equal to

$$v_1 \cdot d(x, x_1)^{-1} + v_2 \cdot d(x, x_2)^{-1} + v_3 \cdot d(x, x_3)^{-1} = \frac{1}{10} + \frac{2}{20} + \frac{3}{30} = \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{3}{10},$$

and the denominator is equal to

$$d(x, x_1)^{-1} + d(x, x_2)^{-1} + d(x, x_3)^{-1} = \frac{1}{10} + \frac{1}{20} + \frac{1}{30} = \frac{6}{60} + \frac{3}{60} + \frac{2}{60} = \frac{11}{60}.$$

Thus, the desired value is equal to

$$v = \frac{\frac{3}{10}}{\frac{11}{60}} = \frac{11 \cdot 10}{3 \cdot 60} = \frac{110}{180} = \frac{11}{18}.$$