Solution to Homework 4

Task. Describe a function $y = 1/(1 + \exp(-x))$ as a composition of invariant functions.

Comment. This function is actively used in neural networks.

Solution. How do we compute this function? First, we compute the first intermediate value

$$z_1 = \exp(-x).$$

In terms of z_1, the desired expression takes the form $y = 1/(1+z_1)$. To compute this expression, next, we add 1 and z_1, thus arriving at the second intermediate result

$$z_2 = 1 + z_1.$$

In terms of z_2, the desired expression takes the form

$$y = 1/z_2.$$

This expression can be directly computed.

So, $y(x)$ is a composition of three functions:

- the first function transforms x into z_1,
- the second function transforms z_1 into z_2, and
- the third function transforms z_2 into y.

By comparing the expressions for these functions with the general expressions for invariant functions, we can see that the first function is shift-scale invariant, the second function is shift-shift invariant, and the third function is scale-scale-invariant.

Thus, the desired function is indeed the composition of invariant functions.