Solution to Homework 5

Task. Describe a function $\sqrt{x^2 + x^4}$ as a scale-invariant combination of two scale-scale-invariant functions.

Hint: use the fact that $x^4 = (x^2)^2$.

Solution. A scale-invariant combination of two functions $y_1(x)$ and $y_2(x)$ has the form $(y_1^p + y_2^p)^{1/p}$. The above function has $1/p = 1/2$, since $a^{1/2} = \sqrt{a}$. Thus, we have $p = 2$.

So, the above function has the form

$$\sqrt{x^2 + x^4} = (x^2 + x^4)^{1/2}.$$

If we take into account that $x^4 = (x^2)^2$, we conclude that it has the form:

$$\sqrt{x^2 + x^4} = (x^2 + (x^2)^2)^{1/2}.$$

This is the desired form for $y_1(x) = x$ and $y_2(x) = x^2$.

Both functions $y_1(x)$ and $y_2(x) = x^2$ are scale-scale-invariant, so we get the desired representation.