

Test 2

1. Why do we sometimes need to consider families of functions instead of a single function?
2. What type of families do we consider? Provide an example of such a family.
- 3–5. Explain, for a family of functions:
 - how shift-invariance leads to functional equations,
 - how functional equations leads to a system of differential equations, and
 - what is the general solution to this system of differential equations.
6. What is the general form of functions from a scale-invariant family? Explain how this leads to a general form of functions from a family which is both shift-invariant and scale-invariant.
7. How can you explain the following empirical formula: $x^2 \cdot (\ln(x))^2$?
8. Explain what is a transformation group, and why the class of all natural transformations should be a finite-parametric transformation group that contains all linear transformations. Prove that the class of all linear functions is a transformation group.
9. Describe all functions from a finite-parametric transformation group that contains all linear transformations.
10. Explain the following empirical dependencies:

$$y = \frac{x}{1+x} \text{ and } y = \frac{1}{1+\exp(-x)}.$$