Test 2, version 2

1–2. Families of functions:

1–2a. Explain why we sometimes need to consider families of functions.

1–2b. What type of families do we consider? Describe a general formula.

1–2c. Provide an example of such a family.

3–5. Explain, for a family of functions:

3. how scale-invariance leads to functional equations,
4. how these functional equations leads to a system of differential equations, and
5. what is the general solution to this system of differential equations.

6. Shift-invariance:

6a. Describe the general form of functions from shift-invariant family.

6b. Explain how this leads to a general form of functions from a family which is both shift-invariant and scale-invariant.

7. Explain the following empirical formula:

$$x^{3.5} \cdot \ln(x) \cdot \sin(5.3 \cdot \ln(x)).$$

8–10. Transformation groups:

8–10a. Explain what is a transformation group.

8–10b. Explain why the class of all natural transformations should be a finite-parametric transformation group that contains all linear transformations.

8–10c. Prove that the class of all power law transformations is a transformation group.

8–10d. Describe all functions from a finite-parametric transformation group that contains all linear transformations.

8–10e. Explain the following empirical dependencies:

$$y = \frac{1-x}{1+2x} \text{ and } y = \frac{1}{1+\exp(-x)}.$$