WIKIPEDIA
25 years of the free encyclopedia

Weighted catenary

A **weighted catenary** (also **flattened catenary**, was defined by William Rankine as **transformed catenary**[1] and thus sometimes called **Rankine curve**[2]) is a catenary curve, but of a special form: while a catenary is the curve formed by a chain under its own weight, a weighted catenary is the curve formed if the chain's weight is not consistent along its length. Formally, a "regular" catenary has the equation

$$y = a \cosh\left(\frac{x}{a}\right) = \frac{a\left(e^{\frac{x}{a}} + e^{-\frac{x}{a}}\right)}{2}$$

for a given value of a. A *weighted catenary* has the equation

$$y = b \cosh\left(\frac{x}{a}\right) = \frac{b\left(e^{\frac{x}{a}} + e^{-\frac{x}{a}}\right)}{2}$$

and now *two* constants enter: a and b.



The Gateway Arch is a weighted catenary: thick at the bottom, thin at the top.

Significance

A freestanding catenary arch has a uniform thickness. However, if

1. the arch is not of uniform thickness,[3]
2. the arch supports more than its own weight,[4]
3. or if gravity varies,[5]

it becomes more complex. A weighted catenary is needed.

The aspect ratio of a weighted catenary (or other curve) describes a rectangular frame containing the selected fragment of the curve theoretically continuing to the infinity. [6][7]



A hanging chain is a regular catenary — and is not weighted.

Examples

The Gateway Arch in the American city of St. Louis (Missouri) is the most famous example of a weighted catenary.

Simple suspension bridges use weighted catenaries.[7]

References

1. Osserman, Robert (February 2010). "Mathematics of the Gateway Arch" (https://www.ams.org/notices/201002/rtx100200220p.pdf) (PDF). *Notices of the American Mathematical Society*. **57** (2): 220–229. ISSN 0002-9920 (https://search.worldcat.org/issn/0002-9920).
2. Andrue, Mario (2020). "The arches of the facade of the Palau Güell. Hyphotesis about its conformation" (https://fundacionantoniogaudi.org/Publicaciones/the-arches-of-the-facade-of-the-palau-guell.pdf) (PDF). *fundacionantoniogaudi.org*. Antonio Gaudi Foundation. Retrieved 5 January 2024.
3. Robert Osserman (February 2010). "Mathematics of the Gateway Arch" (https://www.ams.org/notices/201002/rtx100200220p.pdf) (PDF). Notices of the AMS.
4. Re-review: Catenary and Parabola: Re-review: Catenary and Parabola (http://arch-re-review.blogspot.co.uk/2010/09/catenary-and-parabola.html), accessdate: April 13, 2017
5. MathOverflow: classical mechanics - Catenary curve under non-uniform gravitational field - MathOverflow (https://mathoverflow.net/questions/29247/catenary-curve-under-non-uniform-gravitational-field), accessdate: April 13, 2017
6. Definition from WhatIs.com: What is aspect ratio? - Definition from WhatIs.com (http://whatis.techtarget.com/definition/aspect-ratio), accessdate: April 13, 2017
7. Robert Osserman (2010). "How the Gateway Arch Got its Shape" (http://www.msri.org/people/staff/osserman/papers/NNJ_v12n2_Osserman_pp167-189.pdf) (PDF). Nexus Network Journal. Retrieved 13 April 2017.

External links and references

General links

- One general-interest link (http://arch-re-review.blogspot.com/2010/09/catenary-and-parabola.html)

On the Gateway arch

- Mathematics of the Gateway Arch (https://www.ams.org/notices/201002/rtx100200220p.pdf)
- On the Gateway Arch (http://www.msri.org/people/staff/osserman/papers/NNJ_v12n2_Osserman_pp167-189.pdf)
- A weighted catenary graphed (https://www.desmos.com/calculator/qhvo8xhzq4)

Commons

- Category:Catenary
- Category:Arches

Retrieved from "https://en.wikipedia.org/w/index.php?title=Weighted_catenary&oldid=1310461580"