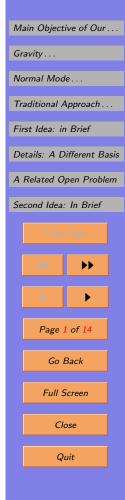
# New Ideas about Joint Inversion (as described in a recent paper on spline techniques)

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# 1. Main Objective of Our Presentation

- Paper: Paula Berkel, Doreen Fischer, and Volker Michel, "Spline multiresolution and numerical results for joint gravitation and normal-mode inversion with an outlook on sparse regularisation", *International Journal of Geomathematics*, August 2010.
- Main objective of the paper: use joint inversion to provide a full 3-D picture of the Earth.
- Geophysically: the authors combine gravity and normal-mode measurements.
- Our applications: we usually deal with a local geophysical analysis.
- Our objective: to describe the main idea that can be used in our applications.

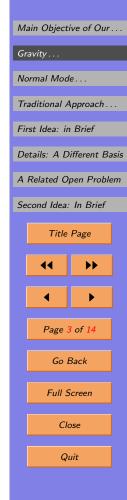


# 2. Gravity Measurements: Non-Uniqueness

• *Ideally:* once we know the gravitational potential field  $\varphi(x)$ , we can determine the density  $\rho(x)$ :

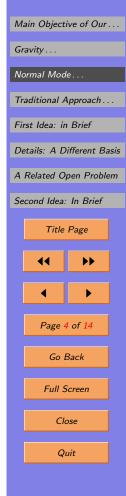
$$\rho = \operatorname{const} \cdot \nabla^2 \varphi(x).$$

- In practice:
  - we only know the values of  $\varphi(x)$  with measurement errors;
  - we only know the values  $\varphi(x)$  in some points x all of which are outside the Earth.
- Result: we cannot uniquely reconstruct the density  $\rho(x)$  ("Earth model") from the gravity measurement results.
- Specifically: several different Earth models  $\rho(x)$  are consistent with the same measurement results.



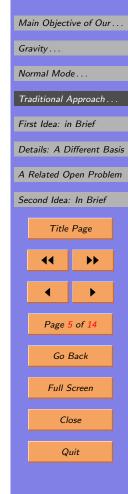
# 3. Non-Uniqueness (cont-d)

- Additional problem: in the isotropic case, the gravity outside the Earth is  $\sim \frac{M}{R^2}$ 
  - this is true when the mass is uniformly distributed inside the Earth;
  - this is true when the mass is mostly concentrated in the center;
  - etc.
- Conclusion:
  - based on measured gravity values,
  - we cannot uniquely determine how density is distributed inside the Earth.
- Thus, to determine an Earth model, we need to supplement gravity data with *other measurements*.



# 4. Need for Error Bounds and for Faster Computations

- Need for error bounds:
  - measurement errors cause errors in the resulting Earth model;
  - it is desirable to find the error bounds on the parameters of the resulting Earth model.
- Need for faster computations:
  - in many data processing techniques, we get values on a dense grid;
  - it often turns out that spatial resolution is not so
    high especially at depth;
  - this means that the difference between  $\rho(x)$  at two neighboring points is not statistically meaningful;
  - it is desirable to save computation time and only generate meaningful values.



#### 5. Normal Mode Measurements

- Gravity measurements provide info about deep layers of Earth.
- To supplement this info, we need to use other geophysical info about such deep layers.
- Such info comes after a strong earthquake, when
  - not only a seismic wave reaches a station,
  - but also it reaches practically the whole Earth and starts oscillations that go on for some time.
- Such post-earthquake oscillations are called *normal mode* oscillations.
- In the paper, gravity measurements are combined with the normal mode measurements.



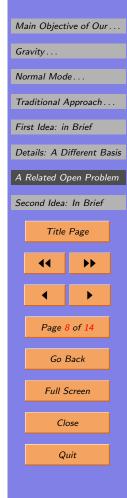
### 6. Traditional Approach to Analyzing Seismic Data

- To understand the new ideas, let us recall the traditional approach.
- First, we reconstruct the velocity (or, equivalently) density model in each 3-D location.
- The model coming out of the computer program has some "features", e.g., areas where:
  - either density is higher than around them,
  - or density is lower than around them.
- Some of these features are real.
- Some are artifacts of the method caused by uncertainty and incompleteness of data.
- One of the ways to distinguish between real features and artifacts is to use a checkerboard method.



### 7. Traditional Approach (cont-d)

- Checkerboard method main idea:
  - we add sinusoidal "checkerboard" patterns to the model,
  - we simulate measurement results corresponding to this perturbed model, and
  - we apply the algorithm to the simulated measurement results.
- Case 1: the algorithm detects the perturbations of this spatial size.
- Conclusion: features of this size in the original model were real.
- Case 2: the algorithm does not detect perturbations.
- Conclusion: features of this small size are artifacts.



#### 8. First Idea: in Brief

- Problem with the traditional approach:
  - we produce a large number of values
  - and then we, in effect, dismiss many of these values as artifacts.
- Fact: an arbitrary function can be approximated by Fourier series

$$\rho(x,y) = \sum_{i=0}^{m} \sum_{j=0}^{n} a_{ij} \cdot \sin(i \cdot x \cdot \omega_0 + j \cdot y \cdot \omega_0 + \varphi).$$

- Traditional approach, in effect: find all the values  $a_{ij}$ , then dismiss most of them.
- New idea: only find the values  $a_{ij}$  that can be (statistically reliably) reconstructed.
- How: we generate  $a_{ij}$  one by one until we get too large a reconstruction error.



#### 9. Details: A Different Basis

- In general:
  - we select a basis  $e_0(x)$ ,  $e_1(x)$ , etc., and
  - we represent an arbitrary function f(x) as a linear combination of the basis functions  $e_n(x)$ :

$$f(x) = \sum_{n=0}^{\infty} a_n \cdot e_n(x).$$

- Examples of bases:
  - polynomials  $1, x, x^2, \ldots$ , resulting in Taylor series;
  - sines, resulting in Fourier series.
- In the above method: we use sines as the basis.
- In general: other bases are possible.
- The paper: uses a combination of polynomials and sines.



### 10. A Related Open Problem

- In the proposed method:
  - if we cannot recover  $a_{ij}$  with a good accuracy,
  - we stop and dismiss all the values corresponding to this spatial resolution.
- *Problem:* this approach does not take into account that accuracy depends on depth.
- In the checkerboard method: for each spatial frequency,
  - we keep shallower features if they can be recovered from the perturbed data, and
  - we dismiss deeper features if they cannot be recovered from the perturbed data.
- Open problem: it is not clear how to do this in the proposed method; maybe use wavelets?



#### 11. Second Idea: In Brief

• Gravity force: an acceleration  $\vec{a}(\vec{x})$  caused at location  $\vec{x}$  by n point bodies with masses  $m_i$  at locations  $\vec{r_i}$ :

$$\vec{a}(\vec{x}) = \sum_{i=1}^{n} \frac{G \cdot m_i}{|\vec{r_i} - \vec{x}|^3} \cdot (\vec{r_i} - \vec{x}).$$

• Continuous case:

$$\vec{a}(\vec{x}) = G \cdot \int \frac{\rho(\vec{r})}{|\vec{r} - \vec{x}|^3} \cdot (\vec{r} - \vec{x}) d\vec{r}.$$

- Fact: this dependence is linear in  $\rho(\vec{r})$ .
- Once we select a basis  $e_{ij}(\vec{r})$ , for  $\rho(\vec{r}) = \sum_{i,j} a_{ij} \cdot e_{ij}(\vec{r})$ , we get

$$\vec{a}(\vec{x}) = G \cdot \sum_{i,j} a_{ij} \cdot \int \frac{e_{ij}(r)}{|\vec{r} - \vec{x}|^3} \cdot (\vec{r} - \vec{x}) \, d\vec{r}.$$

Main Objective of Our...

Gravity . . .

Normal Mode . . .

Traditional Approach...

First Idea: in Brief

Details: A Different Basis

A Related Open Problem

Second Idea: In Brief

Title Page





**>>** 



Go Back

Full Screen

Close

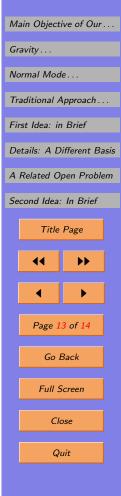
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# 12. Second Idea (cont-d)

- We want to know: how the acceleration  $\vec{a}(\vec{x})$  at a location  $\vec{x}$  is related to the unknown density  $\rho(\vec{r})$ .
- Answer (reminder):  $\vec{a}(\vec{x}) = \sum_{i,j} a_{ij} \cdot I_{ij}$ , where

$$I_{ij} \stackrel{\text{def}}{=} G \cdot \int \frac{e_{ij}(\vec{r})}{|\vec{r} - \vec{x}|^3} \cdot (\vec{r} - \vec{x}) d\vec{r}.$$

- Fact: the values  $I_{ij}$  do not depend on observations, and can thus be pre-computed.
- Resulting idea:
  - we pre-compute the values  $I_{ij}$  before data processing starts;
  - on the data processing stage, we find  $a_{ij}$  by simply solving a system of linear equations.



#### 13. Second Idea: Discussion

- Advantage:
  - we pre-compute the auxiliary values  $I_{ij}$  once, and
  - then use the pre-computed values  $I_{ij}$  to process all the data;
  - thus, we save data processing time.
- Need to take uncertainty into account
  - Fact: we only measure  $\vec{a}$  with some accuracy.
  - Thus: we need to use the Least Squares method to solve the resulting system of linear equations

$$\vec{a}(\vec{x}) \approx \sum_{i,j} a_{ij} \cdot I_{ij}.$$

- For normal mode measurements: the authors use a similar idea – based on linearization.



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