How We Humans Fuse Different Types of Uncertainty when Making Decisions

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Introduction

- In many practical situations, we need to make a decision.
- In many applications, we do not know the exact consequences of each action.
- In such situations, we need to make a decision under uncertainty.
- In many application areas, uncertainty is small – and can be made even smaller by extra measurements.
- For example, for a self-driving car, we can accurately measure all the related values and events.
- However, there are applications when it is difficult to decrease uncertainty.
- One such area is anything related to human activities.
- Humans make individual decisions based on their perceived value of different alternatives.

Decision Theory : A Brief Reminder

- To make a decision, we must:
  - find out the user's preference, and
  - help the user select an alternative which is the best – according to these preferences.
- Traditional approach is based on an assumption that for each two alternatives A' and A'', a user can tell:
  - whether the first alternative is better for him/her; we will denote this by $A'' < A'$;
  - or the second alternative is better; we will denote this by $A' < A''$;
  - or the two given alternatives are of equal value to the user; we will denote this by $A' \sim A''$.

The Notion of Utility

- Under the above assumption, we can form a natural numerical scale for describing preferences.
- Let us select a very bad alternative A_0 and a very good alternative A_1.
- Then, most other alternatives are better than A_0 but worse than A_1.
- For every prob. $p \in [0,1]$, we can form a lottery $L(p)$ in which we get A_1 w/prob. p and A_0 w/prob. $1-p$.
- When $p = 0$, this lottery simply coincides with the alternative A_0: $L(0) = A_0$.
- The larger the probability p of the positive outcome increases, the better the result:

$$p' < p'' \text{ implies } L(p') < L(p'').$$

- Finally, for $p = 1$, the lottery coincides with the alternative A_1: $L(1) = A_1$.
- Thus, we have a continuous scale of alternatives $L(p)$ that monotonically goes from $L(0) = A_0$ to $L(1) = A_1$.
- Due to monotonicity, when p increases, we first have $L(p) < A$, then we have $L(p) > A$.
- The threshold value is called the *utility* of the alternative A:

$$u(A) \stackrel{\text{def}}{=} \sup\{p : L(p) < A\} = \inf\{p : L(p) > A\}.$$

- Then, for every $\varepsilon > 0$, we have

$$L(u(A) - \varepsilon) < A < L(u(A) + \varepsilon).$$

- We will describe such (almost) equivalence by $\equiv$, i.e., we will write that $A \equiv L(u(A))$.

A Rational Agent Should Maximize Utility

- Suppose that we have found the utilities $u(A')$, $u(A'')$, …, of the alternatives A', A'', …
- Which of these alternatives should we choose?
- By definition of utility, we have:
  - $A \equiv L(u(A))$ for every alternative A, and
  - $L(p') < L(p'')$ if and only if $p' < p''$.
- We can thus conclude that A' is preferable to A'' if and only if $u(A') > u(A'')$.
- In other words, we should always select an alternative with the largest possible value of utility.

How to Estimate Utility of an Action

- For each action, we usually know possible outcomes $S_1, \ldots, S_n$.
- We can often estimate the prob. $p_1, \ldots, p_n$ of these outcomes.
- By definition of utility, each situation S_i is equiv. to a lottery $L(u(S_i))$ in which we get:
  - A_1 with probability $u(S_i)$ and
  - A_0 with the remaining probability $1 - u(S_i)$.
- Thus, the action is equivalent to a complex lottery in which:
  - first, we select one of the situations S_i with probability p_i: $P(S_i) = p_i$;
  - then, depending on S_i, we get A_1 with probability $P(A_1 \mid S_i) = u(S_i)$ and A_0 w/probability $1 - u(S_i)$.
- *Reminder:*
  - first, we select one of the situations S_i with probability p_i: $P(S_i) = p_i$;
  - then, depending on S_i, we get A_1 with probability $P(A_1 \mid S_i) = u(S_i)$ and A_0 w/probability $1 - u(S_i)$.
- The prob. of getting A_1 in this complex lottery is:

$$P(A_1) = \sum_{i=1}^{n} P(A_1 \mid S_i) \cdot P(S_i) = \sum_{i=1}^{n} u(S_i) \cdot p_i.$$

- In the complex lottery, we get:
  - A_1 with prob. $u = \sum_{i=1}^{n} p_i \cdot u(S_i)$, and
  - A_0 w/prob. $1 - u$.
- So, we should select the action with the largest value of expected utility $u = \sum p_i \cdot u(S_i)$.

To Practical Applications of Decision Theory

- The numerical value of utility depends on the selection of the alternatives A_0 and A_1.
- If we select a different pair (A_0', A_1'), then utility changes into $u'(A) = a \cdot u(A) + b$ for some $a > 0$ and b.
- The dependence of utility of money is non-linear.
- Utility u is proportional to the square root of the amount m of money $u = c \cdot \sqrt{m}$.
- If we have an amount m of money now, then we can place it in a bank and add an interest.
- So, we get the new amount $m' \stackrel{\text{def}}{=} (1+i) \cdot m$ in a year.
- Thus, the amount m' in a year is equivalent to the value $m = q \cdot m'$ now, where $q \stackrel{\text{def}}{=} 1/(1+i)$.
- This is called *discounting*.

Decision Making Under Interval Uncertainty

- In real life, we rarely know the exact consequences of each action.
- So, for an alternative A, we often only know the bounds on $u(A)$: $\underline{u}(A) \le u(A) \le \overline{u}(A)$.
- For such an interval case, we need to be able to compare the interval-valued alternative with lotteries $L(p)$.
- As a result of such comparison, we will come up with a utility of this interval.
- So, we need to assign, to each interval $[\underline{u}, \overline{u}]$, a utility value $u(\underline{u}, \overline{u}) \in [\underline{u}, \overline{u}]$.
- *Reminder:* utility is determined modulo a linear transformation $u' = a \cdot u + b$.
- *Reasonable to require:* the equivalent utility does not change with re-scaling: for $a > 0$ and b,

$$u(a \cdot u^- + b, a \cdot u^+ + b) = a \cdot u(u^-, u^+) + b.$$

- For $u^- = 0$, $u^+ = 1$, $a = \overline{u} - \underline{u}$, and $b = \underline{u}$, we get

$$u(\underline{u}, \overline{u}) = \alpha_H \cdot (\overline{u} - \underline{u}) + \underline{u} = \alpha_H \cdot \overline{u} + (1 - \alpha_H) \cdot \underline{u}.$$

- This formula was first proposed by a future Nobelist Leo Hurwicz.
- It is known as the Hurwicz optimism-pessimism criterion.

Is "No Trade Theorem" Really a Paradox

- One of the challenges in foundations of finance is the so-called "no trade theorem" paradox:
  - if a trader wants to sell a stock, he/she believes that this stock will go down;
  - however, another trader is willing to buy it;
  - this means that this other expert believes that the stock will go up.
- The fact that equally good experts have different beliefs should dissuade the first expert from selling.
- Thus, trades should be very rare.
- However, in reality, trades are ubiquitous; how can we explain this?

Our Explanation

- Let s be the current cost of the stock. Let m be the mean and σ st. dev. of the (discounted) future gain g.
- Let M be the person's initial amount of money.
- Buying a stock is beneficial if it increases the expected utility, i.e., if $E[\sqrt{M - s + g}] > \sqrt{M}$.
- For small s, this is equivalent to $M > M_0 \stackrel{\text{def}}{=} \dfrac{(m-s)^2 + \sigma^2}{2(m-s)}$.
- So, folks with $M > M_0$ benefit from buying it.
- People with $M < M_0$ benefit from selling it.
- This explains the ubiquity of trading.
- The larger the risk σ, the larger the threshold M_0.
- This explains why depressed people (with lower equivalent value of $M = u^2$) are more risk-averse.

Why Prices for Buying and Selling Objects Are Different

- Intuitively, we should decide, for ourselves, how much each object is worth to us.
- This worth amount should be the largest amount that we should be willing to pay if we are buying this object.
- This same amount should be the smallest amount for which we should agree to sell this objects.
- However, in practice, the buying and selling prices are different.

Our Explanation

- The main reason is that people are not clear on the value of each object.
- At best, they have a range $[\underline{u}, \overline{u}]$ of possible values of this object's worth.
- According to Hurwicz formula, when we buy, we gain the value $u_b = \alpha_H \cdot \overline{u} + (1 - \alpha_H) \cdot \underline{u}$.
- On the other hand, if we already own this object and we sell it, then our loss is between $-\overline{u}$ and $-\underline{u}$.
- The Hurwicz criterion estimates the resulting value as $-u_s$, where $u_s = \alpha_H \cdot \underline{u} + (1 - \alpha_H) \cdot \overline{u}$.
- In the general case, the values u_b and u_s are indeed different.

Explaining "Telescoping Effect" – That Time Perception Is Biased

- People usually underestimate time passed since distant events, and overestimate for recent events.
- Time t is related to utility via discounting: $u = u_0 \cdot q^t$.
- This utility value is always in $[0, u_0]$.
- We only know utility u with some accuracy ε.
- Instead of the original value $u = u_0 \cdot q^t$, we only know that $u \in [u_0 \cdot q^t - \varepsilon, u_0 \cdot q^t + \varepsilon]$.
- For small t, $u_0 \cdot q^t \approx u_0$, so $u_0 \cdot q^t + \varepsilon > u_0$.
- Thus, we have the interval $[u_0 \cdot q^t - \varepsilon, u_0]$, and Hurwicz method leads to the value

$$u(t) = \alpha_H \cdot u_0 + (1 - \alpha_H) \cdot u_0 \cdot (q^t - \varepsilon).$$

- For $t \to 0$, $u_0 \cdot q^t \to u_0$ while $u(t) \to u_0 - (1 - \alpha_H) \cdot \varepsilon < u_0$.
- Thus, for small t, we have $u(t) < u_0 \cdot q^t$.
- The perceived time $\widetilde{t}$ comes from $u(t) = u_0 \cdot q^{\widetilde{t}}$, so $\widetilde{t} > t$.
- For large t, we have $u_0 \cdot q^t - \varepsilon < 0$, so $u \in [0, u_0 \cdot q^t + \varepsilon]$.
- Hurwicz methods leads to the value

$$u(t) = \alpha_H \cdot (u_0 \cdot q^t + \varepsilon).$$

- For $t \to \infty$, $u_0 \cdot q^t \to 0$ while $u(t) \to \alpha_H \cdot \varepsilon > 0$.
- Thus, for large t, we have $u(t) > u_0 \cdot q^t$.
- The perceived time $\widetilde{t}$ comes from $u(t) = u_0 \cdot q^{\widetilde{t}}$, so $\widetilde{t} < t$.
- This explains the telescoping effect.

Future Plans : Theory

- In terms of theoretical analysis, what we have done so far is based on *deterministic* decision making.
- In practice, our decisions are often probabilistic.
- In the same situation, we may select different alternatives, with different probabilities.
- This situation has been analyzed in decision theory by a Nobelist D. McFadden.
- However, his analysis assumes that we know the exact gains related to different alternatives.
- In practice, we usually know the expected gains only with some uncertainty.
- So, our main theoretical research would be to extend McFadden's analysis to the case of uncertainty.

Future Plans : Explanations

- First, there are still seemingly counterintuitive aspects of human behavior that need explaining; e.g.:
  - an often cited phrase that giving is better than receiving
  - seems to be inconsistent with the usual utilitarian models of this behavior.
- Second, the Hurwicz analysis does not explain why some people are more optimistic.
- It is therefore desirable to try to understand this.
- For this purpose, we will analyze which type of behavior works best in different situations.
- Finally, it is desirable to look:
  - not just at the *results* of human decision making,
  - but also at *procedures* that humans use to reach their results.
- For example, as part of these procedures, humans perform some non-traditional approximate computations.
- We plan to analyze how these unusual procedures can be explained by decision making under uncertainty.

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