

Gifted and Talented (G/T): With Others? Separately?

Mathematical Analysis of the Problem

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1. Formulation of the Problem

- Shall we form a separate class with G/T students or keep them with others – while giving them different assignments?
- Possible argument for separate class: gene combinations leading to talents are rare, we need to nurture them.
- But are there math genes that many people lack? Our experience in Russia that most folks can learn math.
- So what prevents many US students from doing well specifically in math?
- Since it is not genes, it must be attitude, i.e., level of enthusiasm.
- Then, attitude can be measured by success in studying.

2. Formulating the Problem in Precise Terms

- Attitudes are contagious: when a student with level x encounters students with levels x_1, \dots, x_n , his/her level changes.
- Let us denote the new level by $x' = f(x, x_1, \dots, x_n)$.

- In the first – linear – approximation,

$$f(x, x_1, \dots, x_n) = a + b \cdot x + c_1 \cdot x_1 + \dots + c_n \cdot x_n.$$

- If there no other students, nothing should change, so $a = 0$ and $b = 1$.
- This value should not depend on how we number students, so we should have $c_1 = \dots = c_n$.
- We want all students to get at least some level of knowledge x_0 .
- A natural goal is to maximize this level, i.e., to maximize

$$x_0 \stackrel{\text{def}}{=} \min(x'_1, \dots, x'_n).$$

- So, we get a precise mathematical problem: how to divide N students with levels x_i into classes of given size n , so as to maximize x_0 .

3. Our Algorithm

- We designed an iterative algorithm for solving this problem.
- In this algorithm, we narrow down the interval $[\underline{x}_0, \bar{x}_0]$ of possible values x_0 , starting with $[\underline{x}_0, \bar{x}_0] = [0, X]$.
- We take x_0 as midpoint of the current interval.
- We match the largest values $x_i > x_0$ with the smallest values $x_j < x_0$.
- We select the proportion of high-enthusiasm students according to the formula

$$h = \frac{x_0 - (1 - c) \cdot x_i - c \cdot n \cdot x_i}{c \cdot n \cdot (x_j - x_i)}.$$

- The n matched students form the first class.
- Then, we match the largest of the remaining values $x_i > x_0$ with the smallest of the remaining values $x_j < x_0$.
- The newly matched n students form the second class, etc.

4. Our Algorithm (cont-d)

- If some students with $x_i < x_0$ are unassigned, we were too optimistic about x_0 , so the new x_0 -interval is $[\underline{x}_0, x_0]$.
- If some students with $x_i > x_0$ are unassigned, we were too pessimistic about x_0 , so the new x_0 -interval is $[x_0, \bar{x}_0]$.
- We repeat the above procedure with the new midpoint x_0 .
- If needed, we continue these iterations – until everyone is assigned.
- How do we know that it works?
- In Russia, in each class, the best-performing students were matched with the worst-performing ones, and it worked well.