A Constrained Multi-Objective Optimization Framework for Multiple Geophysical Data Sets

An Investigation of Crust & Upper Mantle Complexity

Lennox Thompson[1], Aaron A. Velasco[1], Vladik Kreinovich [1], & Anibal Sosa[2]

(1) University of Texas at El Paso, El Paso, TX, United States

(2) Universidad Icesi, Cali, Colombia

Contact Information:
Department of Geological Sciences
University of Texas at El Paso
500 West University Avenue
lethompson@miners.utep.edu
Poster # T43-4706

Introduction

- Combining multiple datasets will help us to better determine physical properties of the Earth structure.
- Use an optimization scheme to jointly invert multiple datasets to obtain 3D shear wave models.
- The 3D models will help us characterize and interpret the crust and upper mantle structures of regions like Texas.

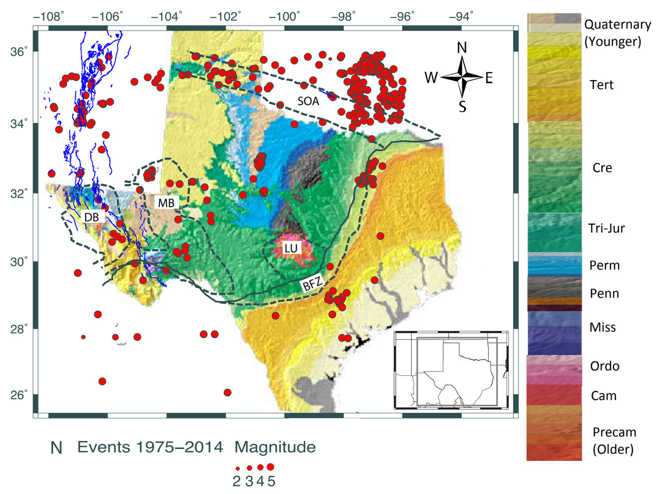

Fig 1: *Tectonic map of Texas with dark colors represent older rock material & light colors represent younger material*

Receiver Function (RF)

- RFs are a time series representation of the Earth's response relative to wave propagation near a recording station.
- Postive or negative spike amplitudes represent postive or negative seismic velocity constrast of RFs.

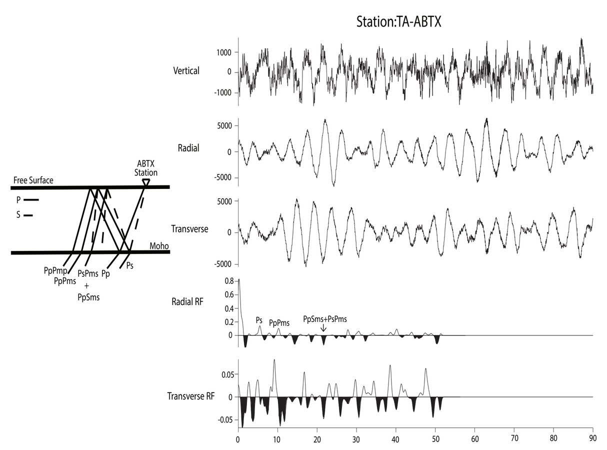

Fig 2: *(Right) Components of the waveform for station ABTX and the radial and transverse RFs. The halfspace represents usually the Moho. (Left) The Moho converted phase Ps and the multiples, along with their ray paths.*

Surface Wave Dispersion (SW)

- SW in general differ from body waves in many respects
- Travel slower, lower frequencies, largest amplitudes, and their velocities are in fact dependent on frequency
- SW dispersion curves from SLU (**Herrmann**, *http://www.eas.ulu.edu/eqc/eqc_research/NATOMO*)

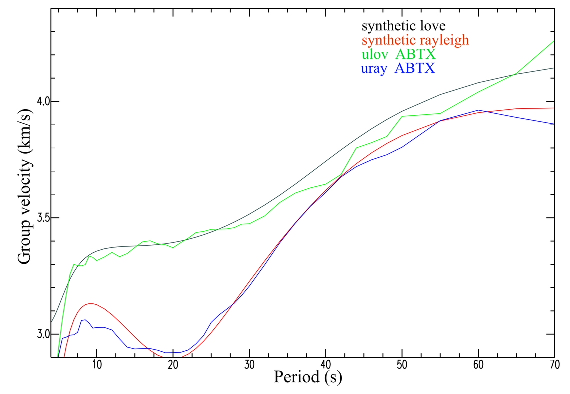

Fig 3: *Rayleigh and Love group velocity curves for station ABTX.*

Travel Times (TT)

- Travel Times acquired from Array Network Facility (ANF) catalog
- Travel times are considered a nonlinear inverse problem given the relationship between the measured data (travel times) and the unknown model parameters (the velocity field).

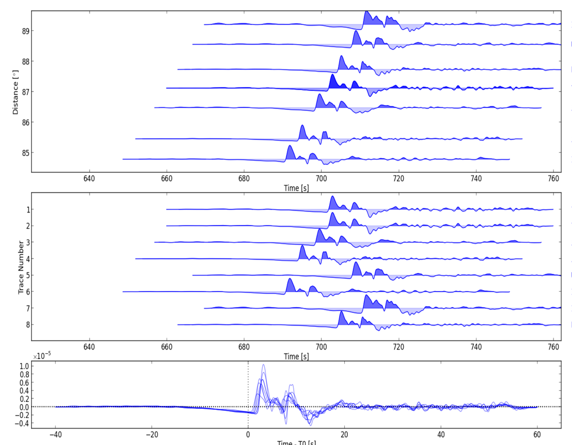

Fig 4: *(First) plot shows P-wave travel times (Dist (deg) vs. Time(sec)) for different USArray Stations that were used for this research.*

Methods

Forward Problem

If we know the layered shear velocity distribution $x=(x_1,\ldots,x_n)$ at n different horizontal layers, then we can evaluate the measured quantities $y=(x_1,\ldots,x_m)$ (e.g., the travel times) by applying an appropriate nonlinear operator $F(x)$ that uses the velocities x to predict the Earth's response $y=F(x)$;

$$F(x)=(F_1(x),\ldots,F_m(x))\in \mathbb{R}^m, x=(x_1,\ldots,x_n)\in \mathbb{R}^n \; (m\gg n) \quad (1)$$

The operator F relates the data space and the model space. In other words, if we know the velocity model x, then we can predict the Earth's response based on the velocity model.

Inverse Problem

Given an observed data vector, $y\in \mathbb{R}^m$,we want to find the unknown model, x, such that $F(x)$ approximates as much as possible.

$$\min_x ||F(x)-y||^2 \quad (2)$$

where

$$F(x)=W\begin{pmatrix}F^{SW}(x)\\ F^{RF}(x)\\ F^{TT}(x)\end{pmatrix}\in \mathbb{R}^m,$$

$$y=W\begin{pmatrix}y^{SW}\\ y^{RF}\\ y^{TT}\end{pmatrix}\in \mathbb{R}^n$$

We can then use a first order Taylor approximation of the operator F around some suitable model $\bar{x}_k$:

$$F(x)\cong F(\bar{x}_k)+F'(\bar{x}_k)\Delta x=F(\bar{x}_k)+F'(\bar{x}_k)(x-\bar{x}_k), \quad (3)$$

where $F'(\bar{x}_k)$ is the matrix formed by the partial derivatives of F. Therefore, we rewrite the problem (2) as

$$\begin{aligned}\min_x \; & \frac{1}{2}\left\|F'(\bar{x}_k)x+r(\bar{x}_k)\right\|^2 \\ s.t \; & g(\bar{x}_k)-s=0 \\ & s\geq 0\end{aligned} \quad (4)$$

where $s\in \mathbb{R}^{2n}$ is a slack variable, $r(\bar{x}_k)=F(\bar{x}_k)-y-F'(\bar{x}_k)\bar{x}_k$, and $g(x)$ is a vector of constraints, including constraints $x_i-a_i\geq 0$ and $b_i-x_i\geq 0$ that describe the bounds $a_i\leq x_i\leq b_i$ on velocities x_i at different layers. Problem (4) can be solved using the Primal-Dual Interior-Point method [Sosa, 2013].

Pareto Front

- Generate solutions corresponding to all possible combinations of actual accuracies
- The generation of solutions from combinations of weights, define the Pareto front corresponding to multi-objective optimization problem

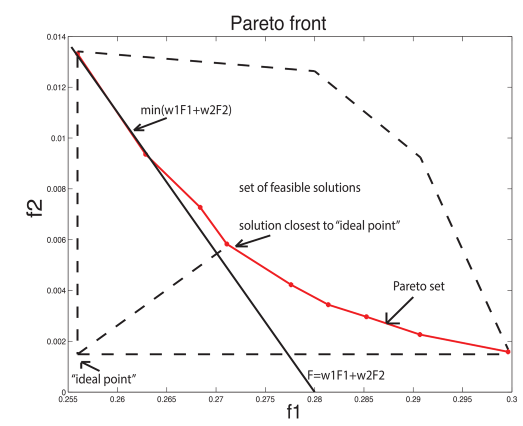

Fig 5: *Illustration of the solution set or Pareto front, which is, defined as the weights times the perspective objective functions.*

Data

- Earthscope Transportable USArray Data. 70 km nominal spacing & 400 seismic stations migrating from west coast to east coast.
- In Texas, Transportable Array migrated from the west to the east between 2008-2010.

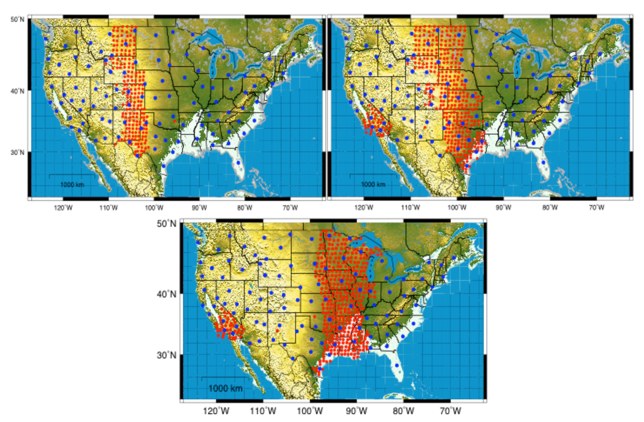

Fig 6: *All USArray Stations migrating from west to east Texas during 2009-2011.*

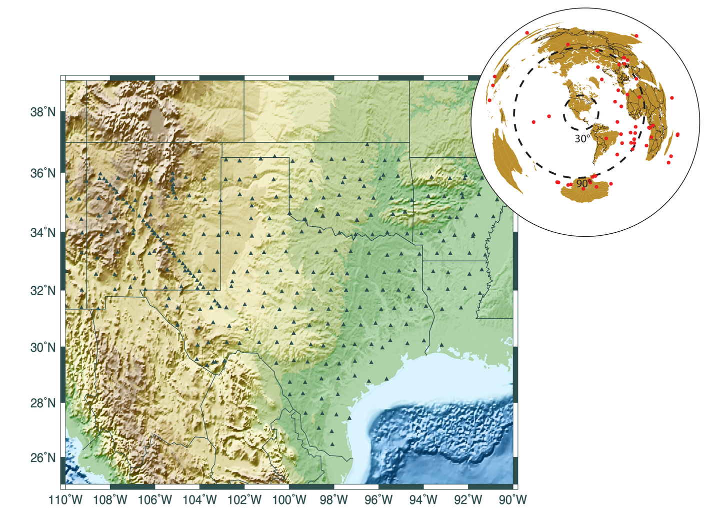

Fig 7: *All USArray Stations within Texas (Left) and all epicenter locations of events used for this research (Right).*

Results

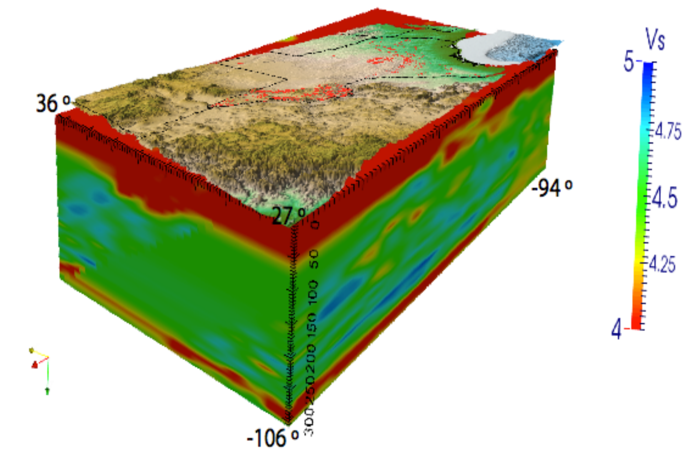

Fig 8: *3D Shear wave model of Texas (Left) and Shear wave models of the upper crust (Right).*

Future Work

- We will incorporate the fourth geophysical dataset (*Gravity*) into our optimization scheme.
- Need to compute the average of all shear wave models with Gravity included in the inversion scheme.
- To compute the level of uncertainty & error when we introduce the fourth geophysical data (Gravity) into our inversion.

References

Sosa, A., A.A. Velasco, L. Velasquez, M. Argaez, and R. Romero. (2013). Constrained Optimization framework for joint inversion of geophysical data sets. Geophys. J. Int., 195, pp. 197–211.

Acknowledgements

Ezer Patlan from the Department of Geological Sciences and **Dr. R. Romero** from the Department of Computer Science.This work is being funded by NSF CREST Grant Cyber-ShARE HRD-0734825.